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Zurich UZH

**ETH**

Eidgenössische Technische Hochschule Zürich  
Swiss Federal Institute of Technology Zurich

KING'S  
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LONDON

# Beyond Nyquist – Accelerating Magnetic Resonance Imaging

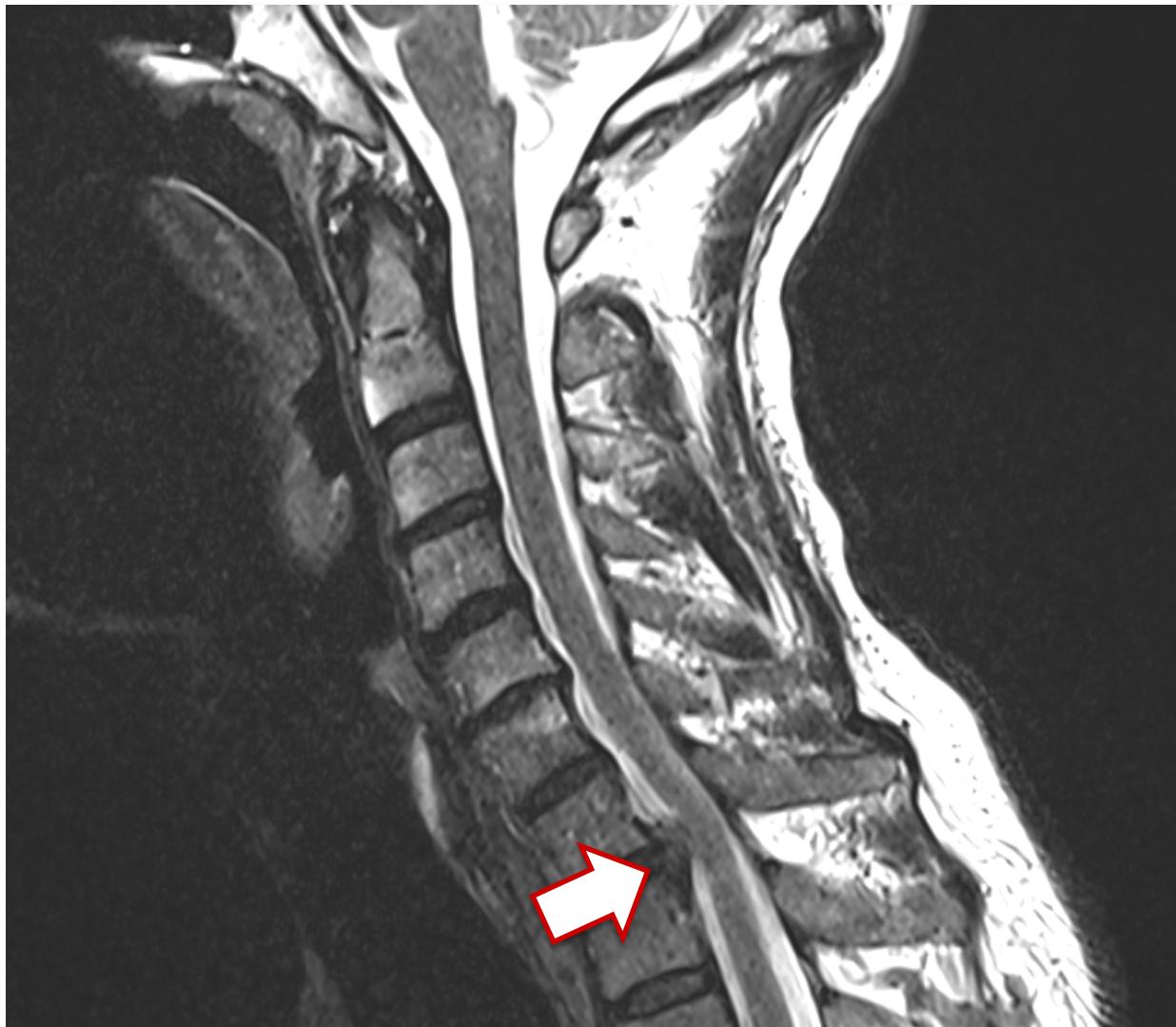
Sebastian Kozerke

Institute for Biomedical Engineering, University and ETH Zurich, Switzerland

Imaging Sciences and Biomedical Engineering, King's College London, UK

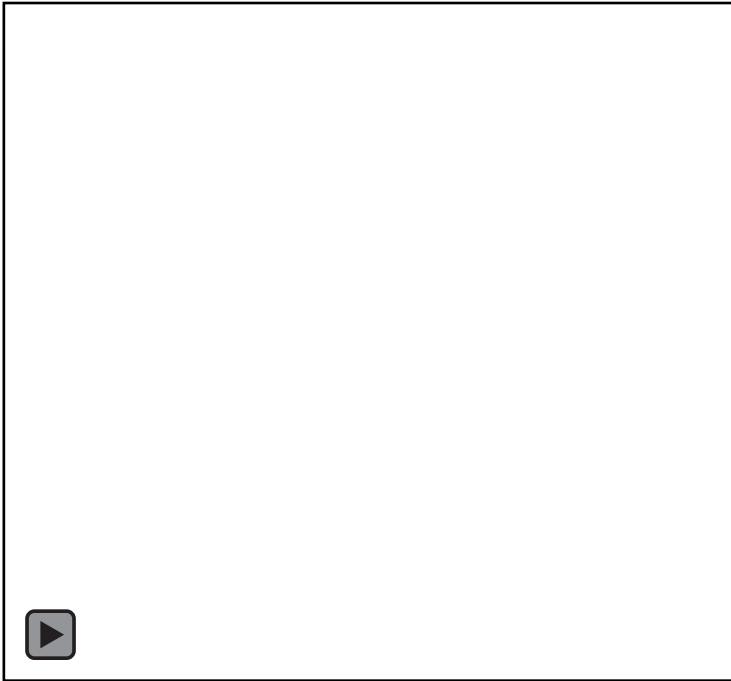


It's useful...

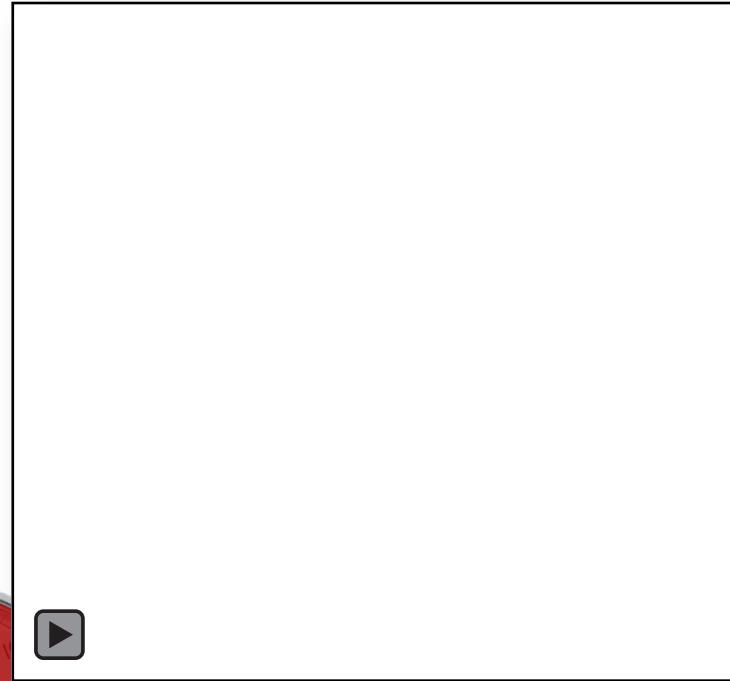


# Blood flow quantification

Healthy volunteer



Patient with dilated aorta



7D encoding ( $x-y-z-v_x-v_y-v_z-t$ )  
 $2 \times 2 \times 2 \text{ mm}^3$



# Magnetic Resonance Imaging

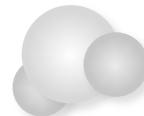
Radio transmitter



**Position → Frequency**



Signal source



Magnetic field



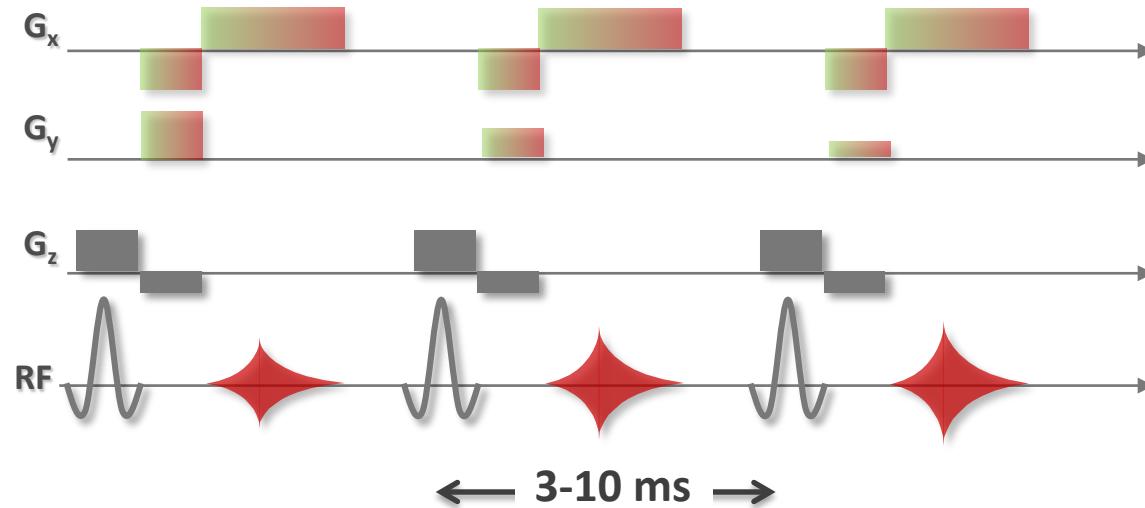
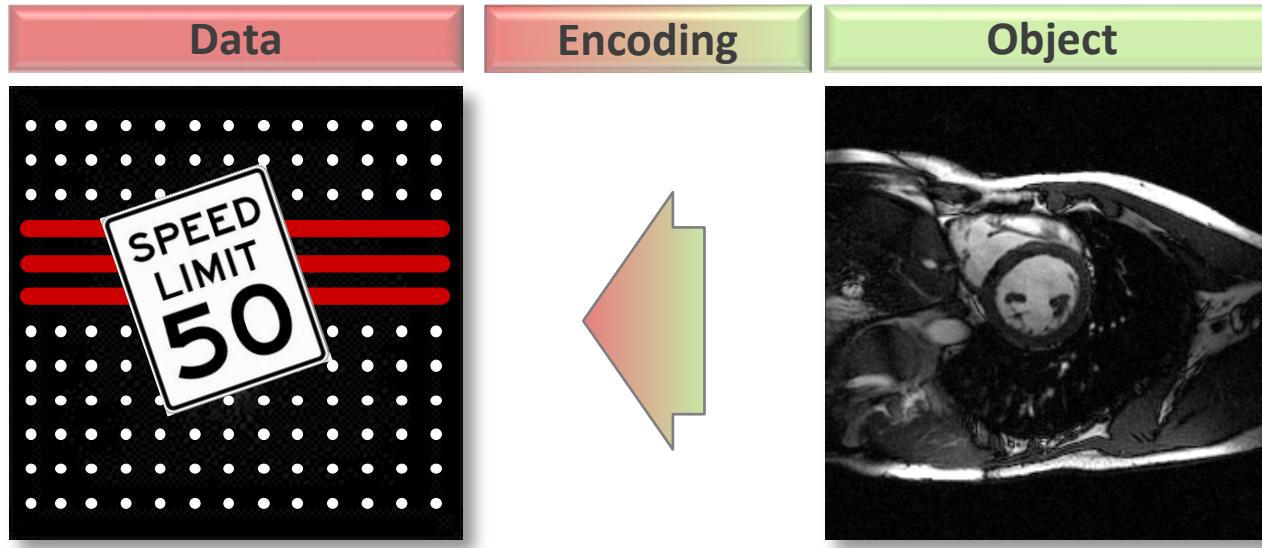
Radio receiver



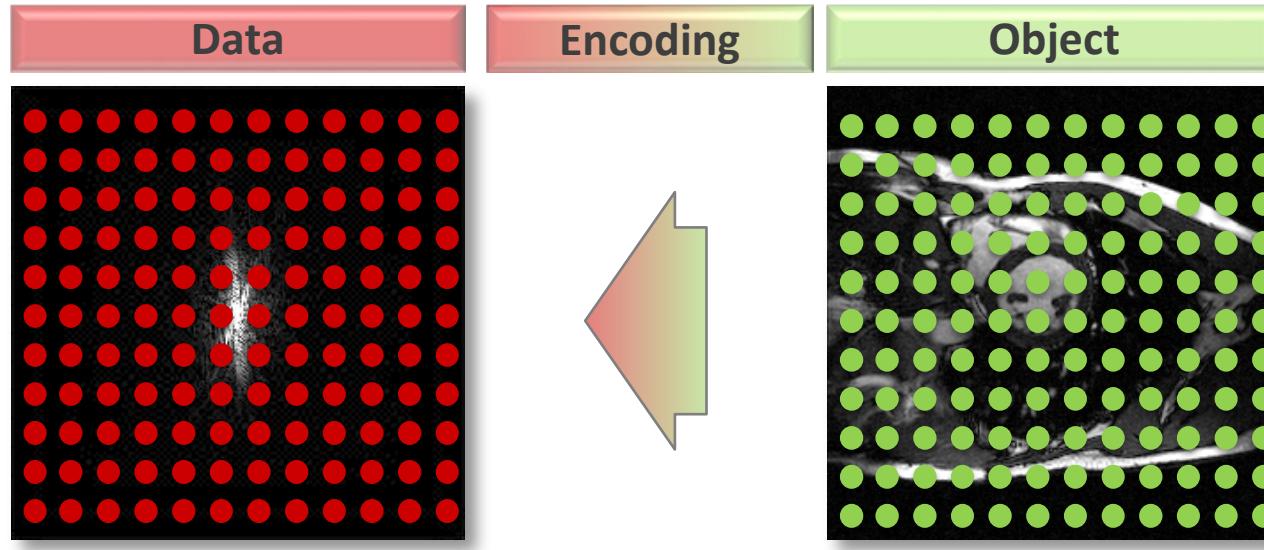
**Frequency → Position**



# Encoding



# Encoding



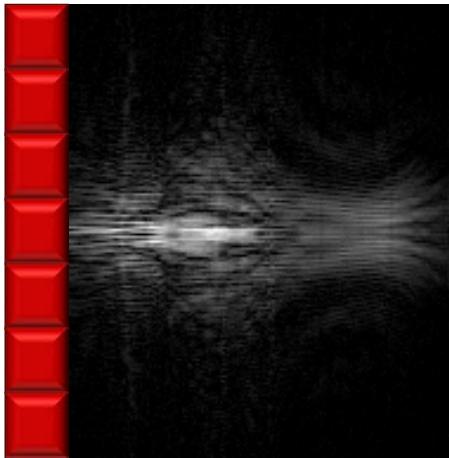
$$d_{\kappa} = \int e^{\vec{j}\vec{k}_{\kappa} \cdot \vec{x}} \rho(\vec{x}) d\vec{x}$$

$$d_{\kappa} = \sum_{\xi} e^{\vec{j}\vec{k}_{\kappa} \cdot \vec{x}_{\xi}} \rho(\vec{x}_{\xi})$$

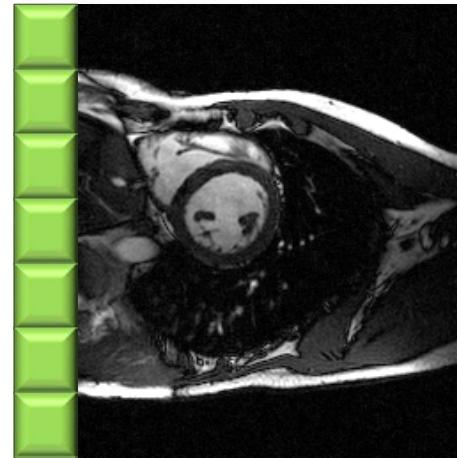
$$\vec{d} = E \vec{\rho}$$

# Decoding

Data



Image



Encoding

$$\vec{d} = \mathbf{E} \cdot \vec{\rho} + \vec{\eta}$$



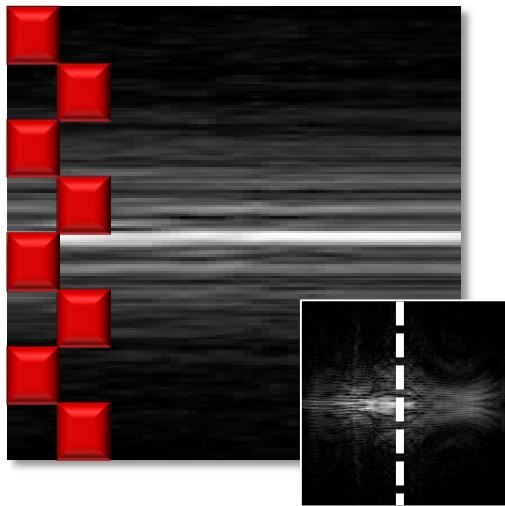
Decoding

$$\vec{i} = (\mathbf{E}^H \Psi^{-1} \mathbf{E})^{-1} \mathbf{E}^H \Psi^{-1} \cdot \vec{d}$$

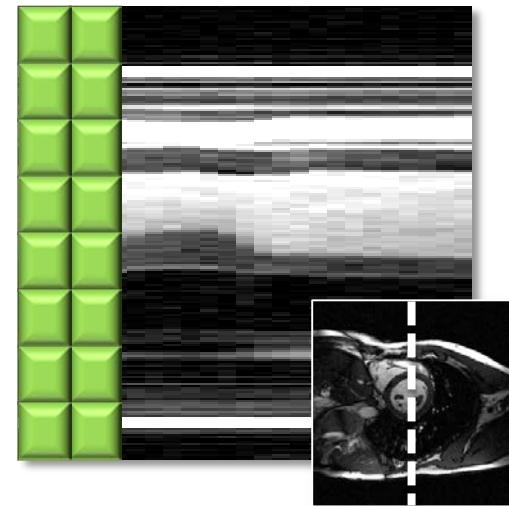
$$\vec{i} \rightarrow \operatorname{argmin}_i (\vec{d} - \mathbf{E} \cdot \vec{i})^H \Psi^{-1} (\vec{d} - \mathbf{E} \cdot \vec{i})$$

# Linear decoding

Data



Image



$$\vec{i} \rightarrow$$

$$\operatorname{argmin}_i \|\vec{d} - E \cdot \vec{i}\|_2^2$$

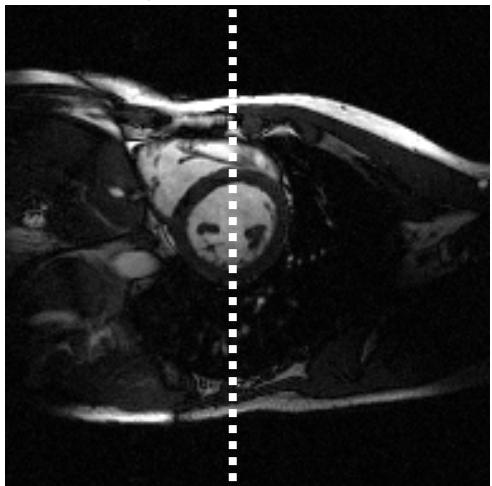
$$\vec{i} \rightarrow$$

$$\operatorname{argmin}_i \|\vec{d} - E \cdot \vec{i}\|_2^2 + \lambda \|\Theta^{-1} \cdot \vec{i}\|_2^2$$

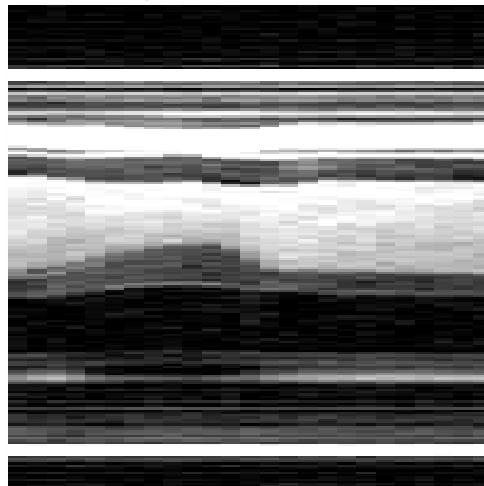
Prior info

# Transform coding

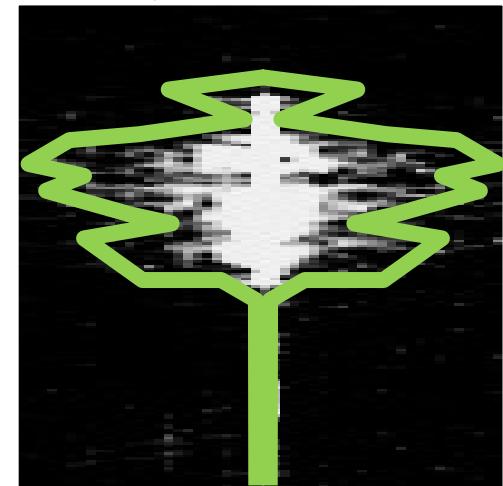
x-t space



x-t space



x-f space

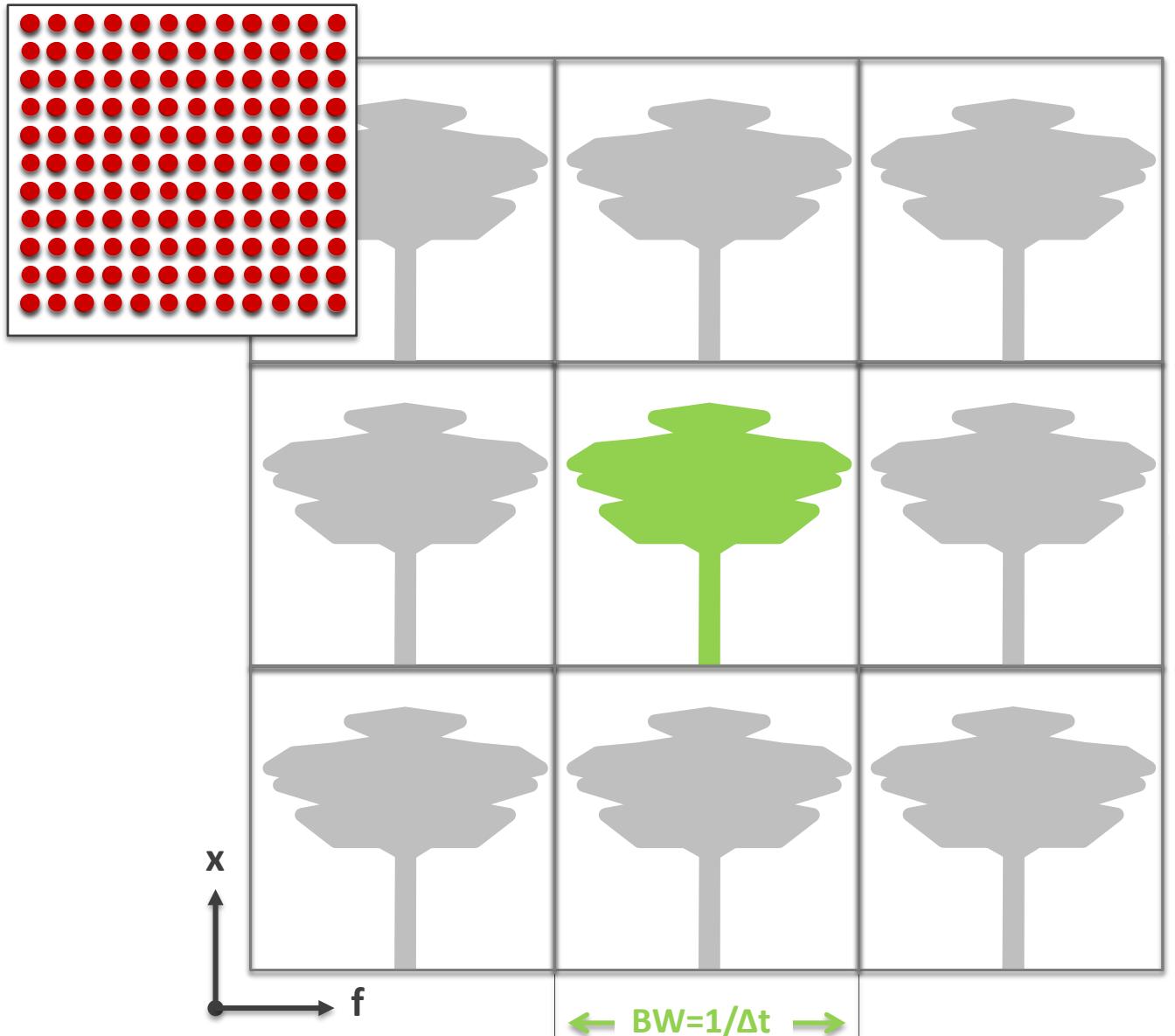


$$\tilde{p}(x, f_\phi) = \sum e^{j2\pi f_\phi t_\tau} p(x, t_\tau)$$

# Transform coding

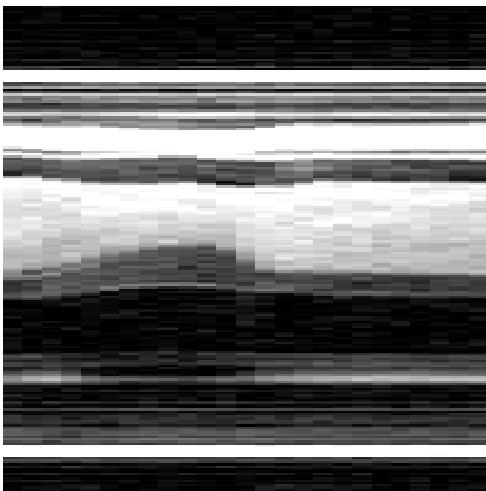


Harry Nyquist  
1889-1976

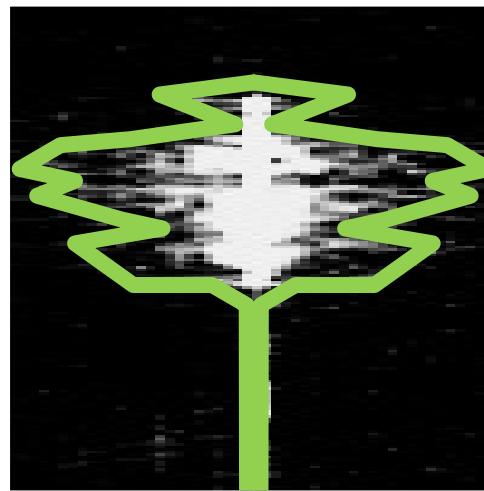


# Transform coding

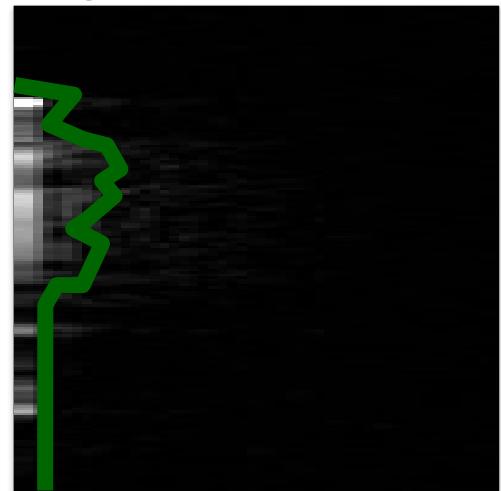
x-t space



x-f space



x-pc space



x  
↑  
f

x  
↑  
pc

$$\tilde{p}(x, f) = U \Sigma V^H = \sum_{n=1}^{pc} w(x, n) b(n, f)$$

# Linear decoding

Temporal regularization

$$\vec{i} \rightarrow \operatorname{argmin}_i \| \vec{d} - E \cdot \vec{i} \|_2^2 + \lambda \| \Phi_{t:t} \vec{i} \|_2^2$$

Spatiotemporal regularization

$$\vec{i} \rightarrow \operatorname{argmin}_i \| \vec{d} - E \cdot \vec{i} \|_2^2 + \lambda \| \Phi_{x,t} \vec{i} \|_2^2$$

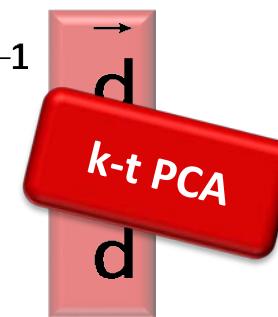
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Spatiotemporal training ( $R < N_c$ )

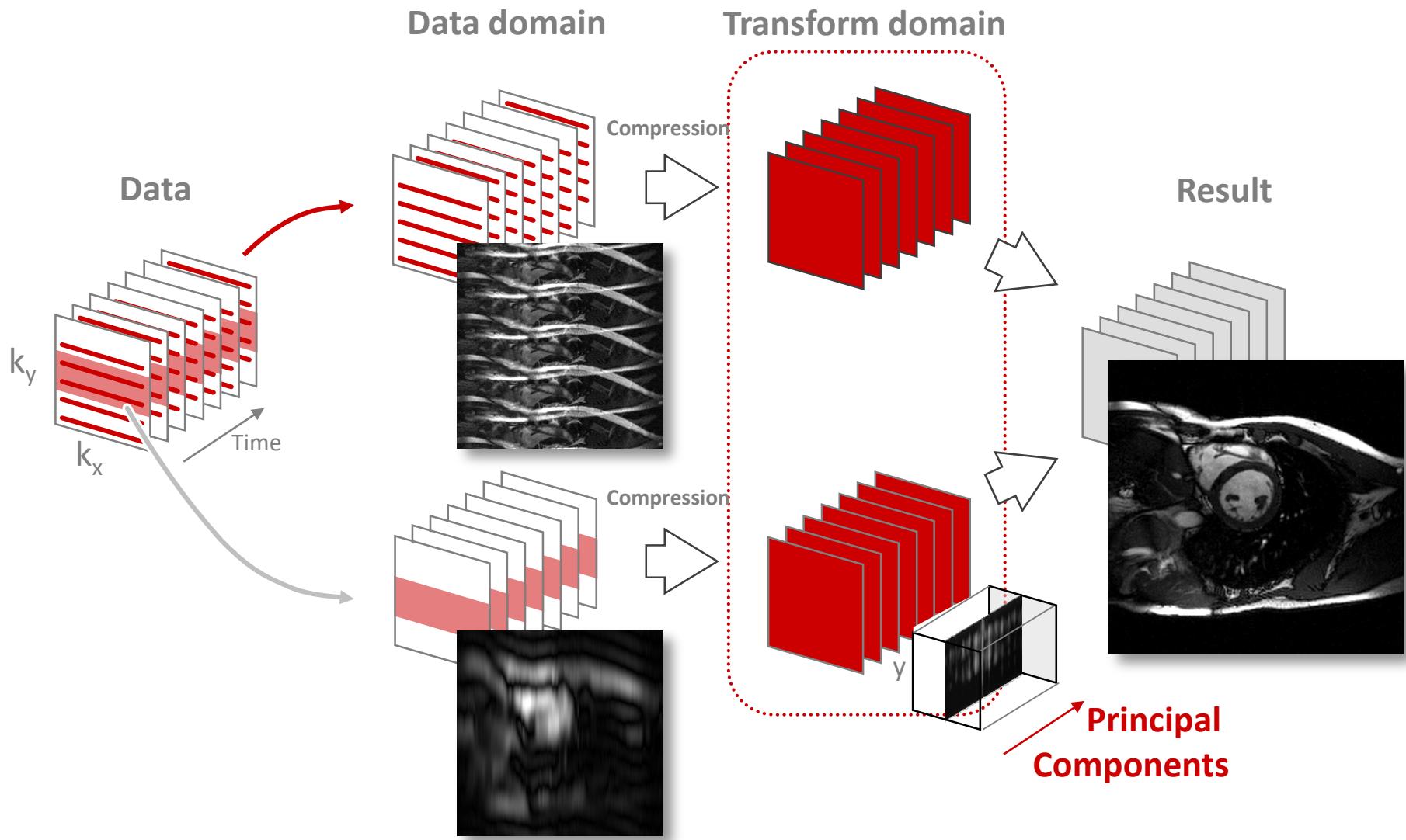
$$\vec{w} = (\tilde{E}^H \tilde{\Psi}^{-1} \tilde{E} + \lambda \tilde{\Theta}^{-1})^{-1} \tilde{E}^H \tilde{\Psi}^{-1} \vec{d}$$

Spatiotemporal training ( $R > N_c$ )

$$\vec{w} = \tilde{\Theta}^H (\tilde{E}^H \tilde{\Theta}^H + \lambda \tilde{\Psi})^{-1} \vec{d}$$



# k-t PCA



# Nonlinear decoding

Temporal regularization

$$\vec{i} \rightarrow \operatorname{argmin}_i \|\vec{d} - E \cdot \vec{i}\|_2^2 + \lambda \|\Phi_t \vec{i}\|_2^2$$

Spatiotemporal regularization

$$\vec{i} \rightarrow \operatorname{argmin}_i \|\vec{d} - E \cdot \vec{i}\|_2^2 + \lambda \|\Phi_{x,t} \vec{i}\|_2^2$$

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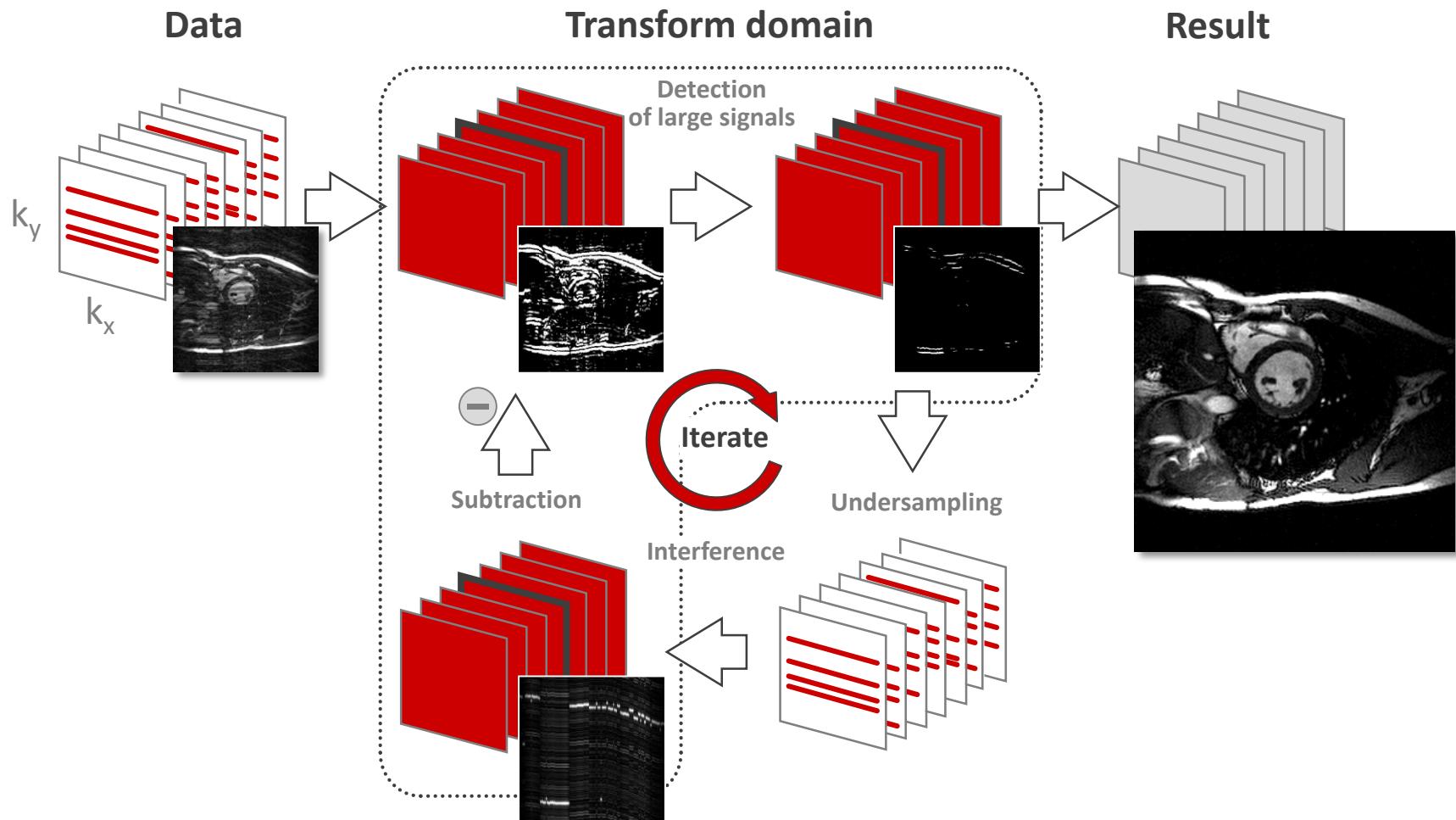
Compressed Sensing

$$\vec{i} \rightarrow \operatorname{argmin}_i \|\vec{d} - E \cdot \vec{i}\|_2^2 + \lambda \|\Phi_{x,f} \vec{i}\|_1$$

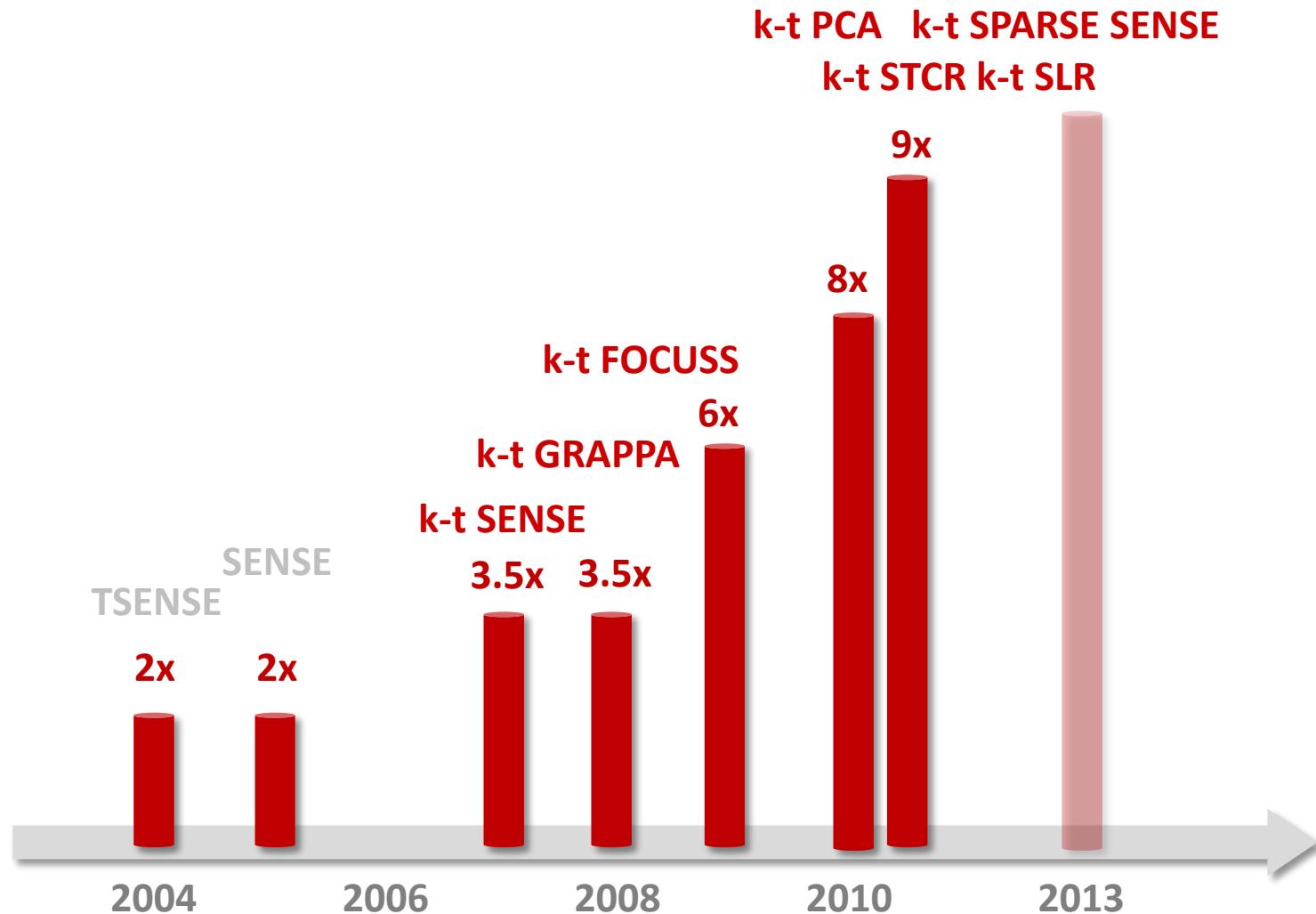
Low-rank sparse decomposition

$$\vec{i} \rightarrow \operatorname{argmin}_i \|\vec{d} - E \cdot (L + S)\|_2^2 + \lambda \|L\|_* + \lambda \|S\|_1$$

# Compressed Sensing



# Undersampling progress – Cardiac



Kellman P et al. MRM 2004

Plein S et al. Radiology 2005

Plein S et al. MRM 2007

Jung B et al. JMRI 2008

Nayak KS et al. JCMR 2008

Vitanis V et al. MRM 2010

Otazo R et al. MRM 2010

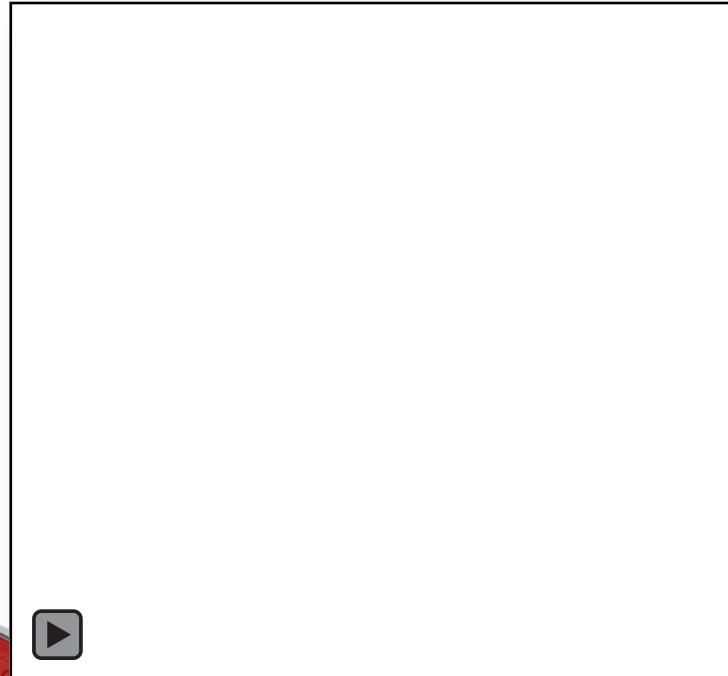
Lingala SG et al. IEEE 2011

# Blood flow quantification

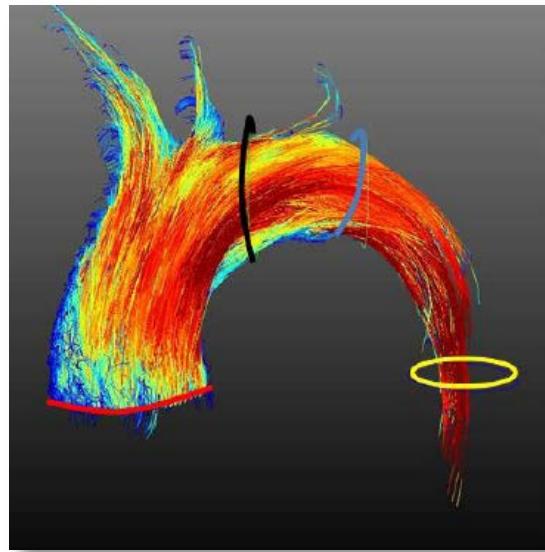
Healthy volunteer



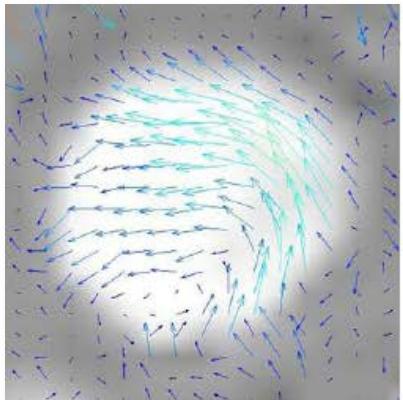
Patient with dilated aorta



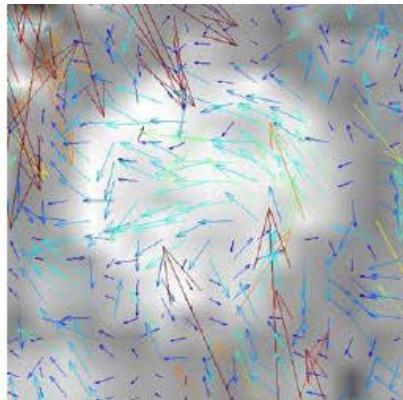
# Reconstruction accuracy



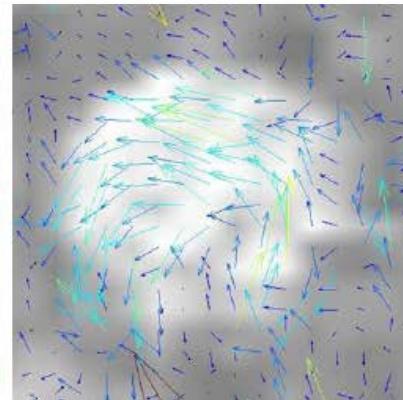
**R=1**



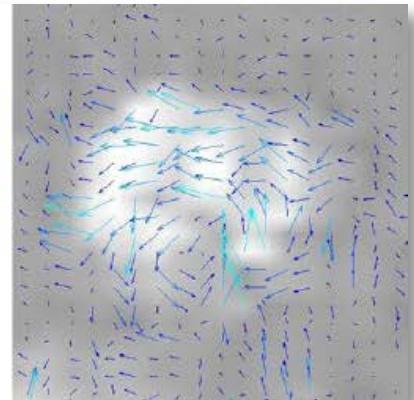
**R=4**



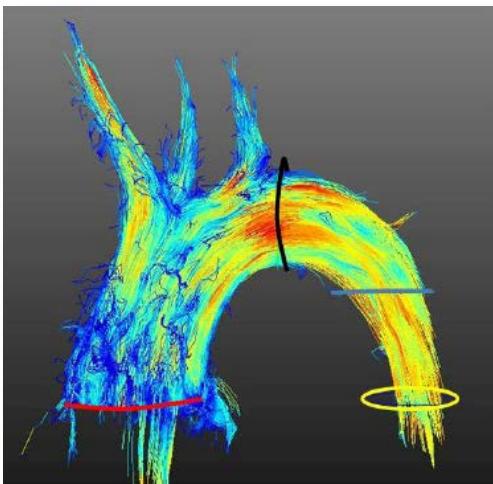
**R=8**



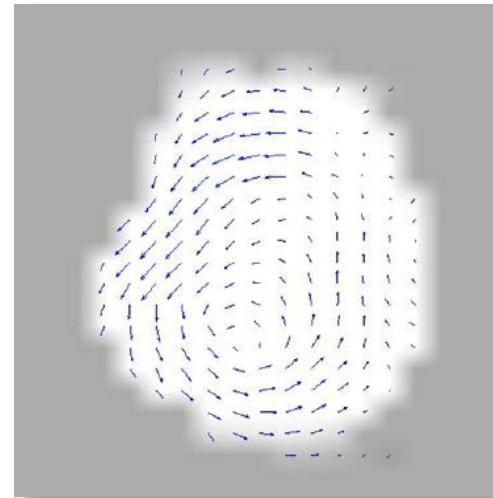
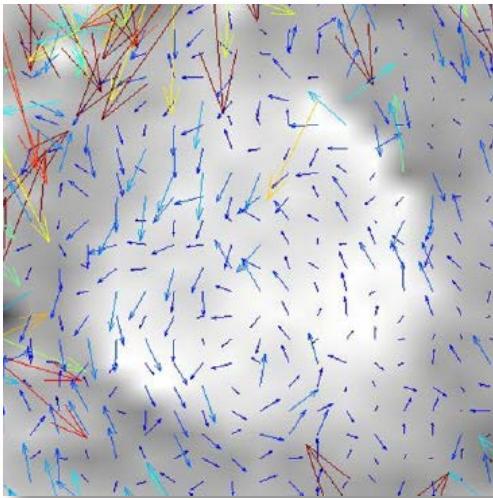
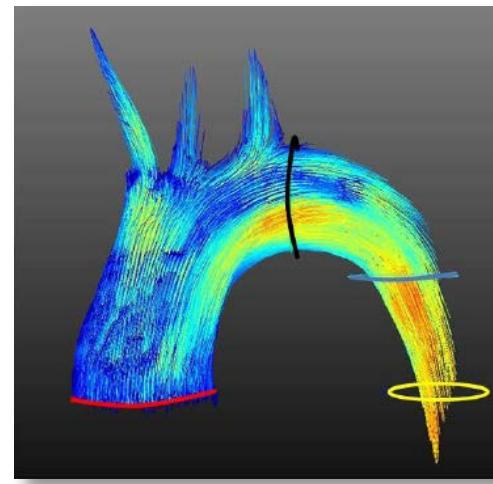
**R=16**



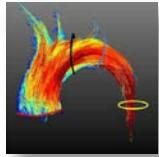
# Divergence-free image reconstruction



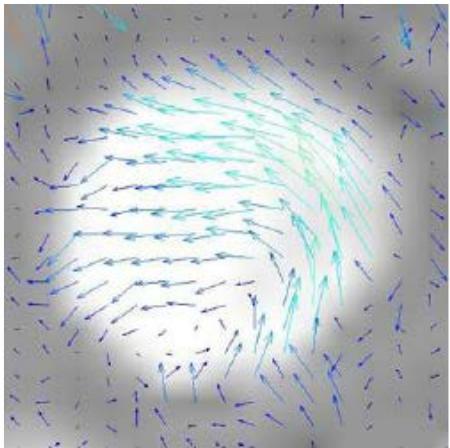
$$\text{div}(\mathbf{v})=0$$



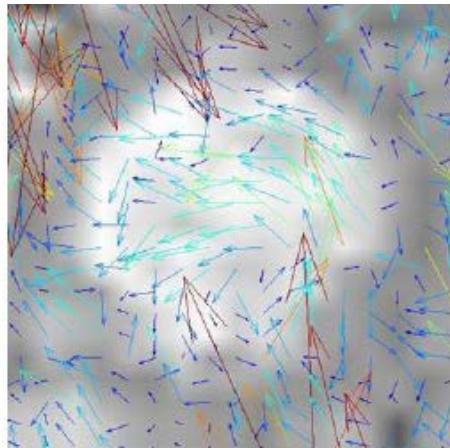
# Divergence-free image reconstruction



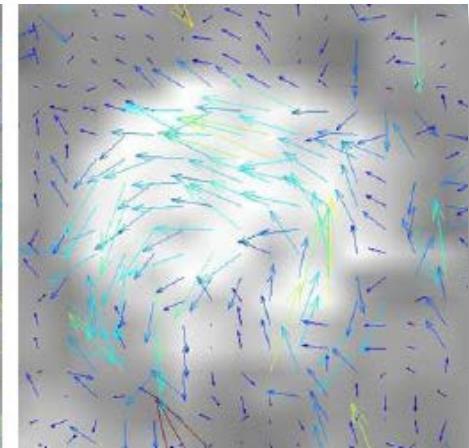
**R=1**



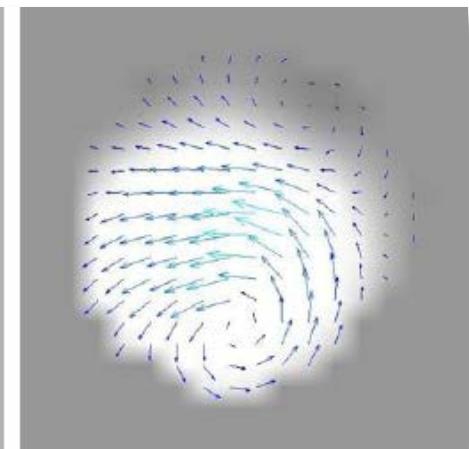
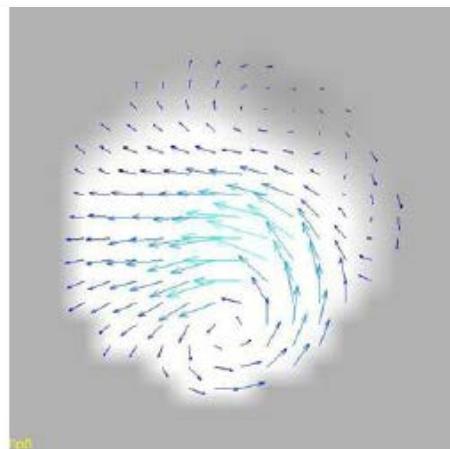
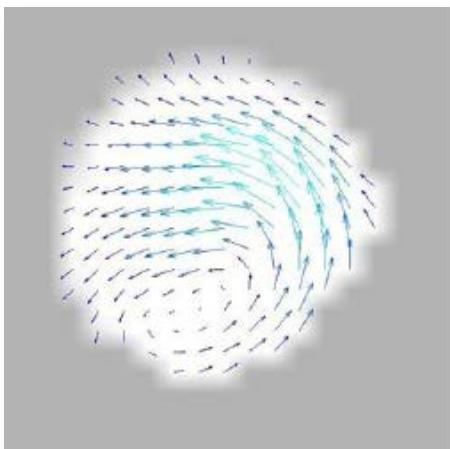
**R=4**



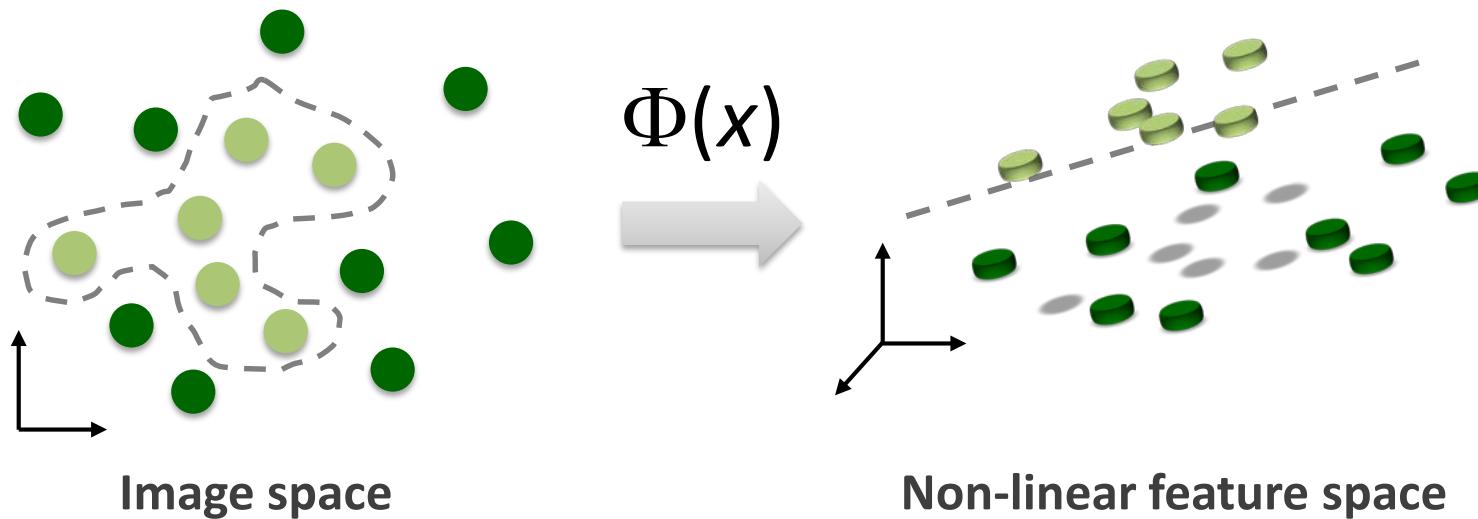
**R=8**



$$\text{div}(\mathbf{v})=0$$



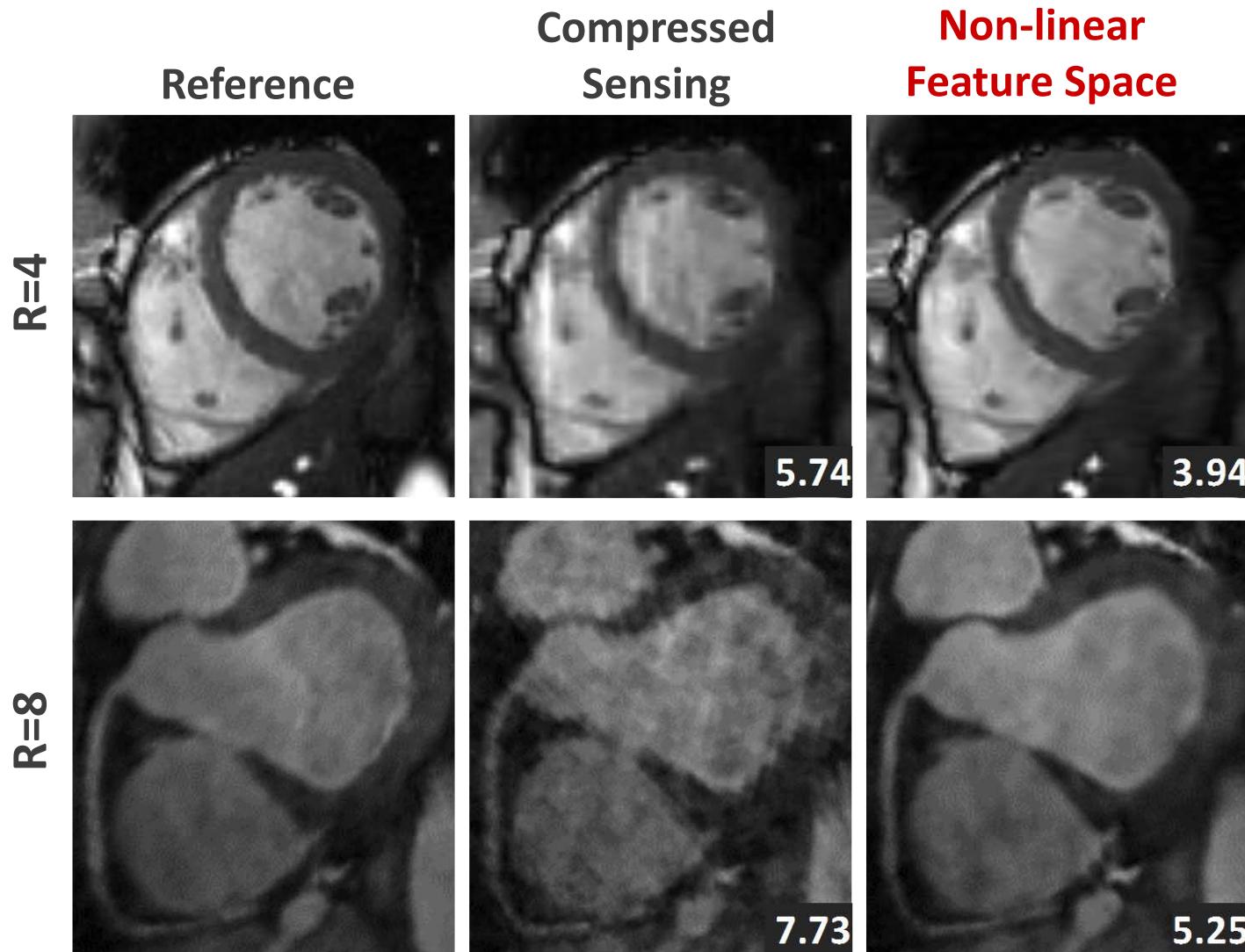
# Image reconstruction in non-linear feature spaces



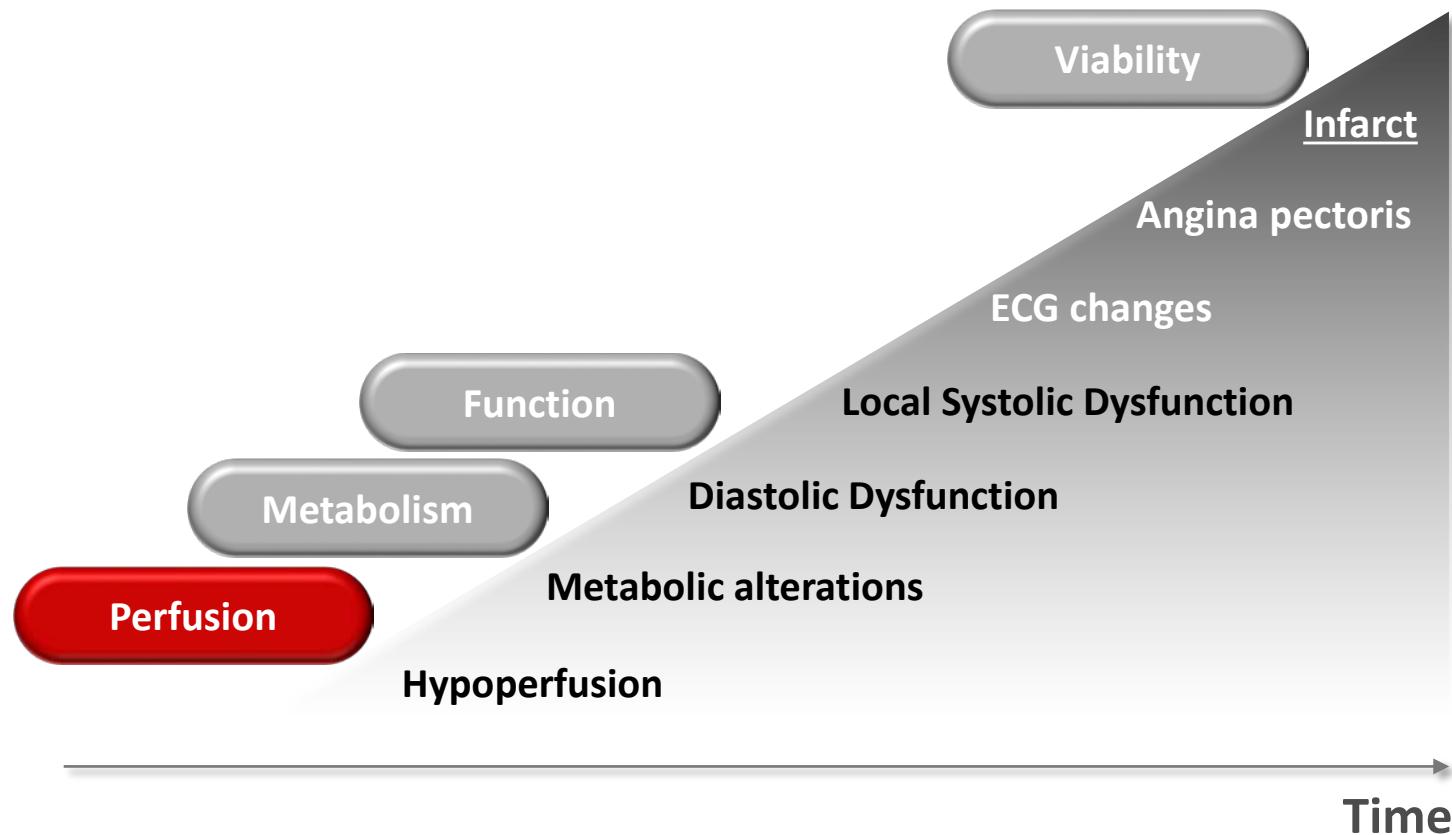
$$\vec{i} \rightarrow$$

$$\vec{i}_{n+1} = \mathcal{P}_{\text{kPCA}}(\vec{i}_n - E^H(E\vec{i}_n - \vec{d}))$$

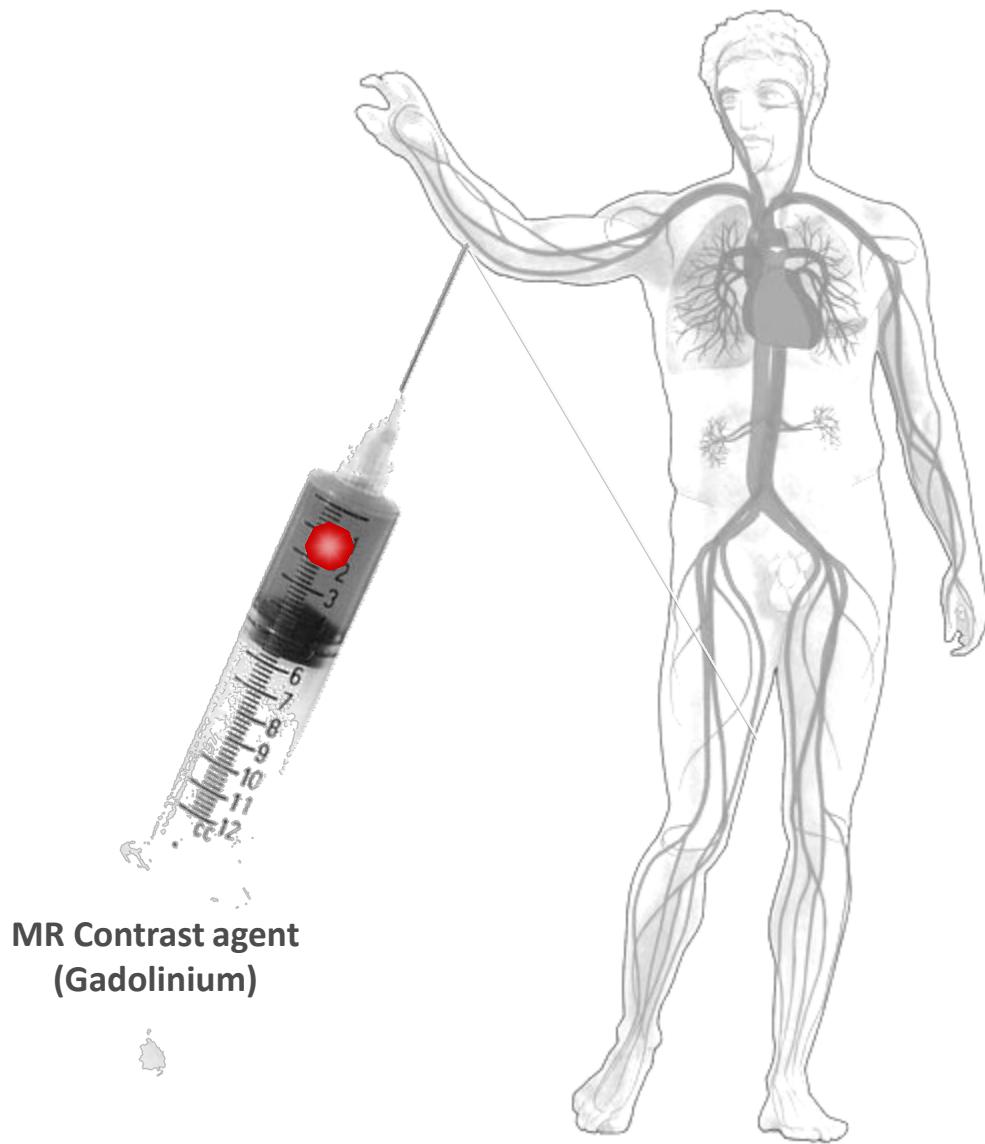
# Image reconstruction in non-linear feature spaces



# Ischemic cascade



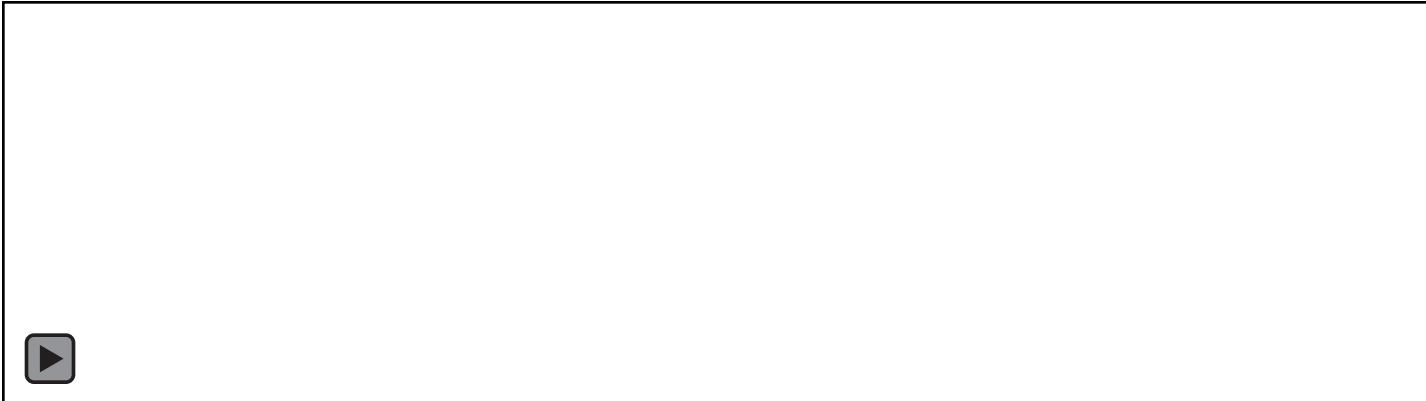
# 3D dynamic perfusion imaging



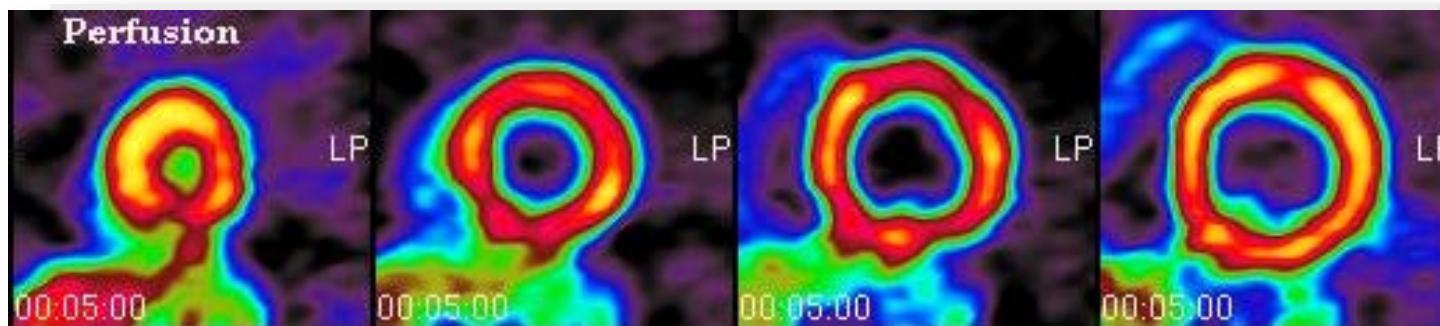
# 2D Perfusion imaging



**5x 2D k-t SENSE (1.1 x 1.1 mm<sup>2</sup>)**

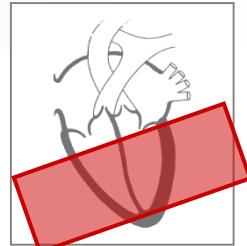


## Stress PET

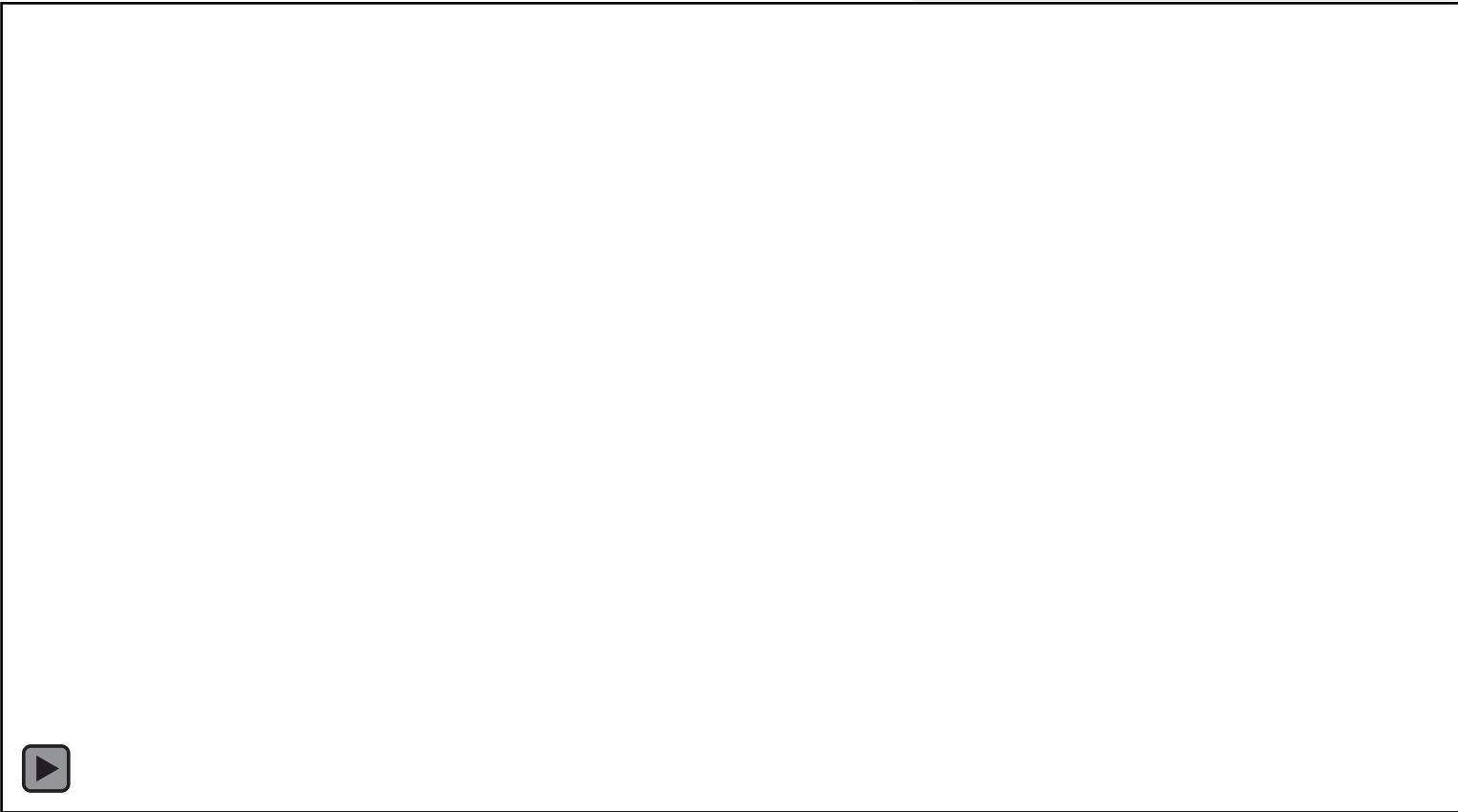




## k-t PCA



**10x k-t PCA** (2.2x2.2x10 mm<sup>3</sup>)



# 3D Perfusion trial

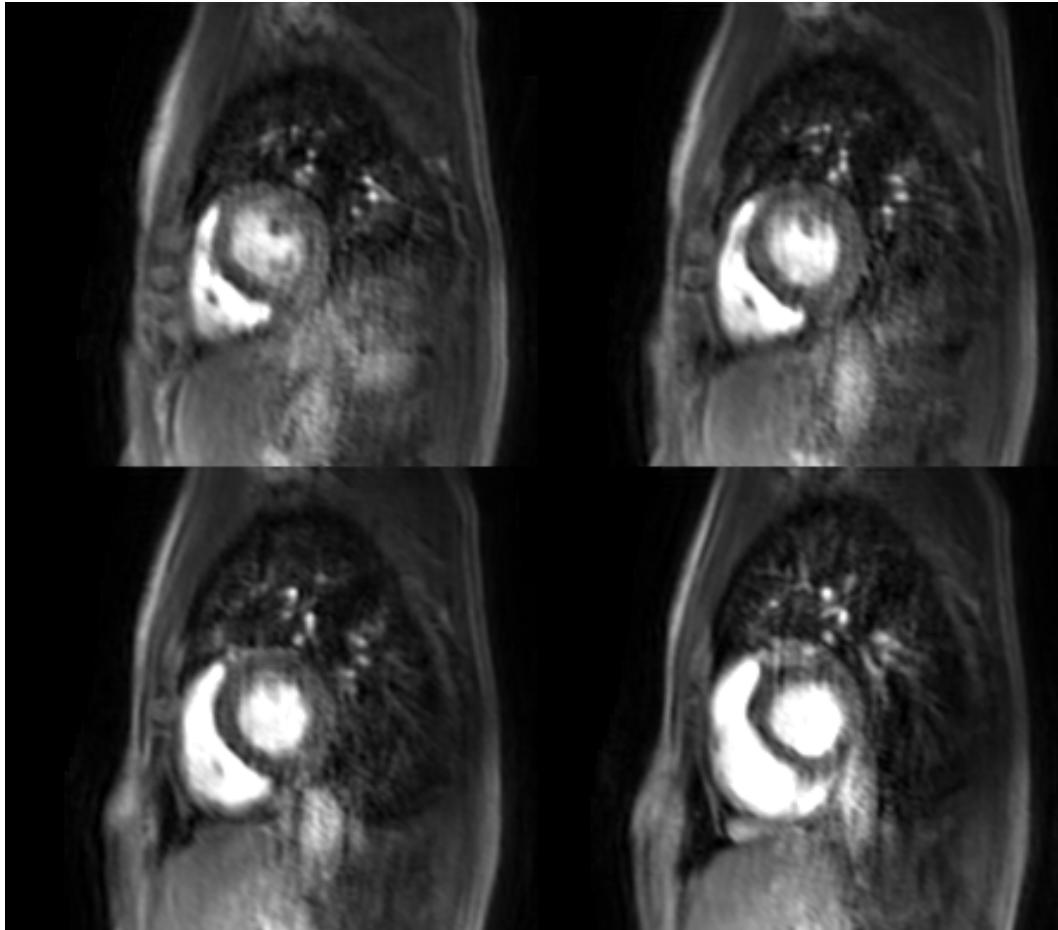
## 3D Perfusion Multi-centre study

| N=150           | CMR vs. FFR | CMR vs. QCA |
|-----------------|-------------|-------------|
| Sensitivity (%) | 90          | 77          |
| Specificity (%) | 89          | 94          |
| NPV (%)         | 88          | 68          |
| PPV (%)         | 89          | 96          |

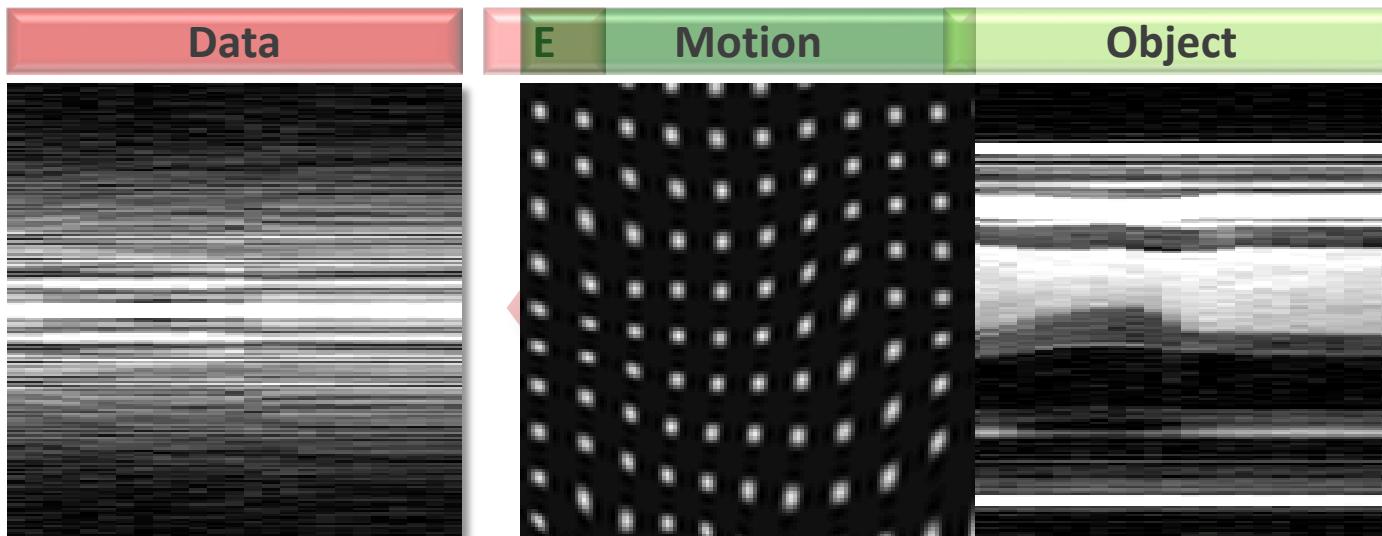


# Motion artifacts

**10x 3D k-t PCA** ( $2.2 \times 2.2 \text{ mm}^2$ )



# Generic matrix description of motion

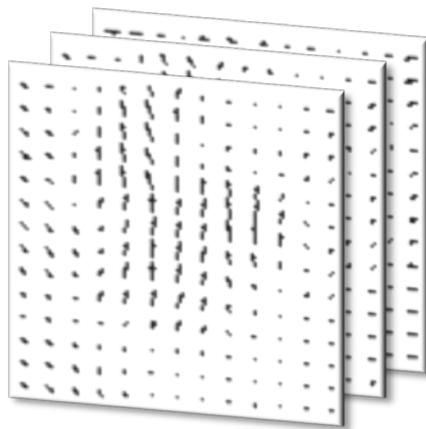


$$\vec{d} = \sum_i \underbrace{\begin{matrix} E_i \\ T_i \\ \vec{\rho} \end{matrix}}_{\hat{E}}$$

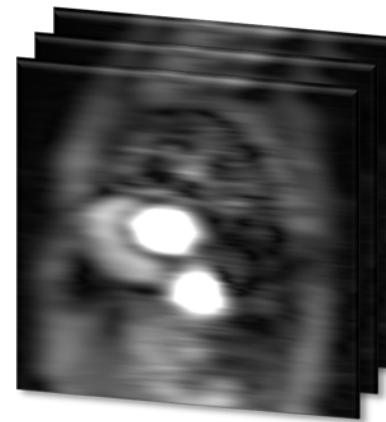
$$\vec{i} = \left( \hat{E}^H \Psi^{-1} \hat{E} + \lambda \Theta^{-1} \right)^{-1} \hat{E}^H \Psi^{-1} \vec{d}$$

# Data-driven motion correction

Motion



Estimate ( $x-t$ )



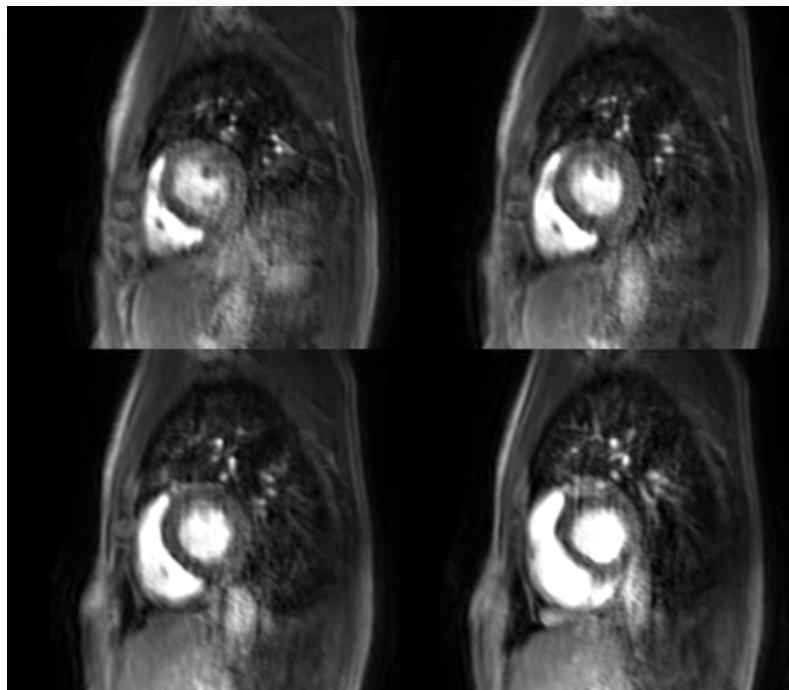
$\vec{i} \rightarrow$

$$\text{argmin}_i \left\| \vec{d} - E \cdot \vec{i} \right\|_2^2 + \lambda \left\| (\mathbf{T}_{x-pc} \Theta)^{-1} B_{f \rightarrow pc} E_{t \rightarrow f} \vec{T}i \right\|_2^2$$

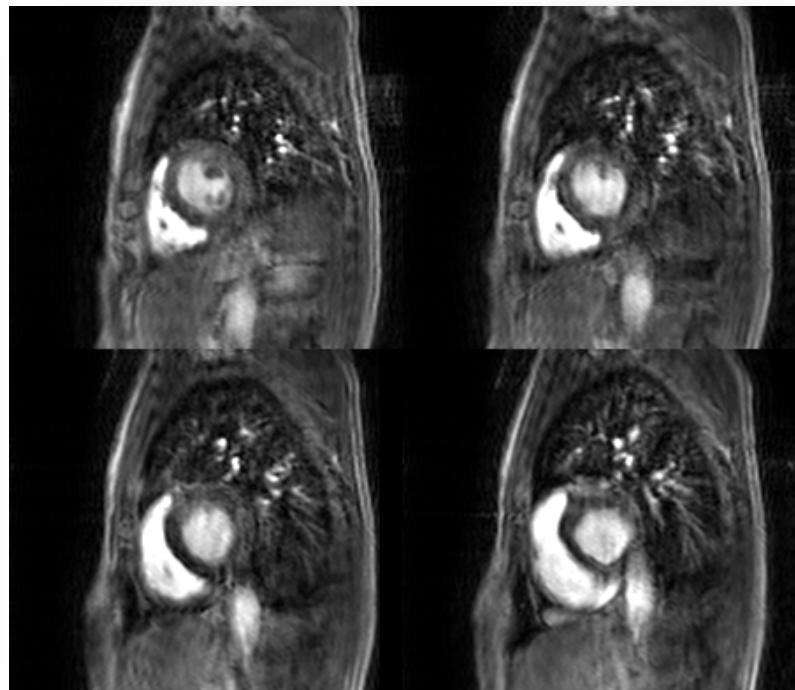
k-t PCA<sup>mc</sup>

# Data-driven motion correction

10x 3D k-t PCA

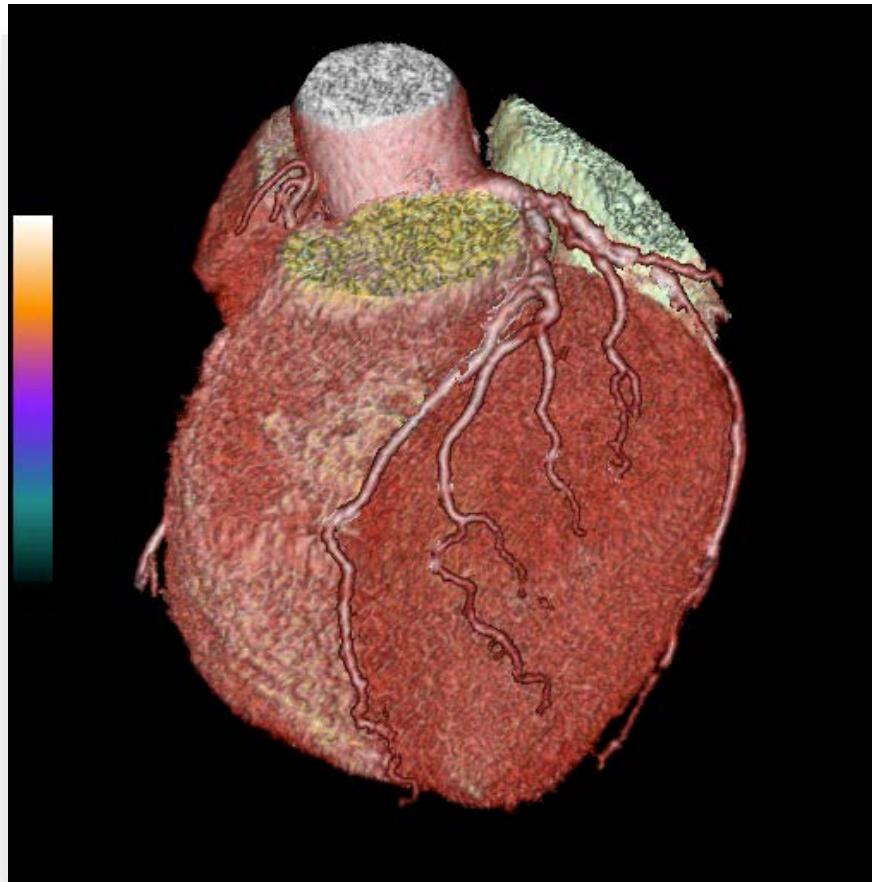


10x 3D k-t PCA<sup>mc</sup>

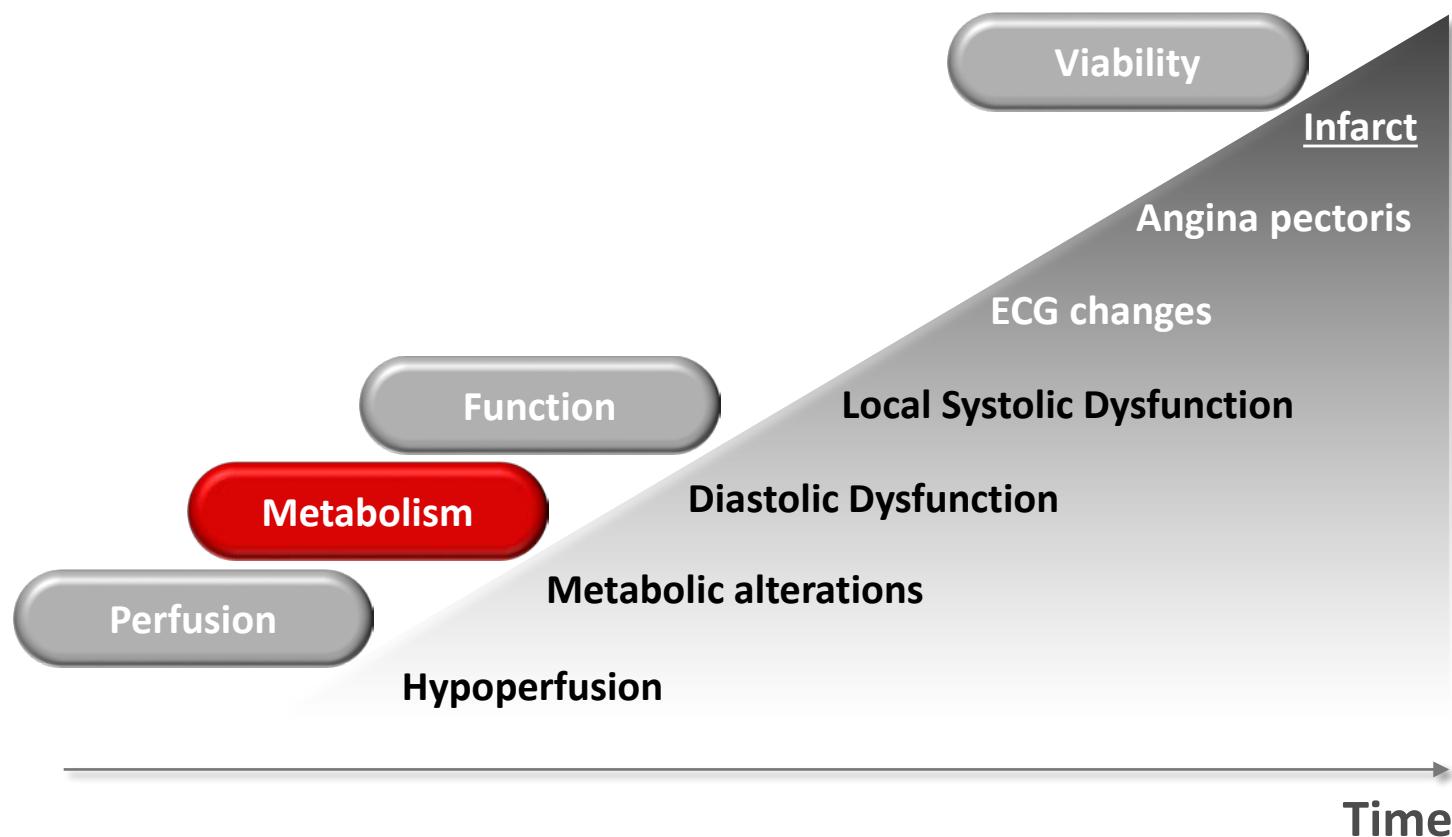


# Hybrid imaging – MR/CT

**3D MR Perfusion + CT Angiography**



# Ischemic cascade



# Chemical shift

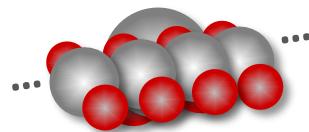
Radio transmitter



Chemical Shift → Frequency



Signal source



Magnetic field



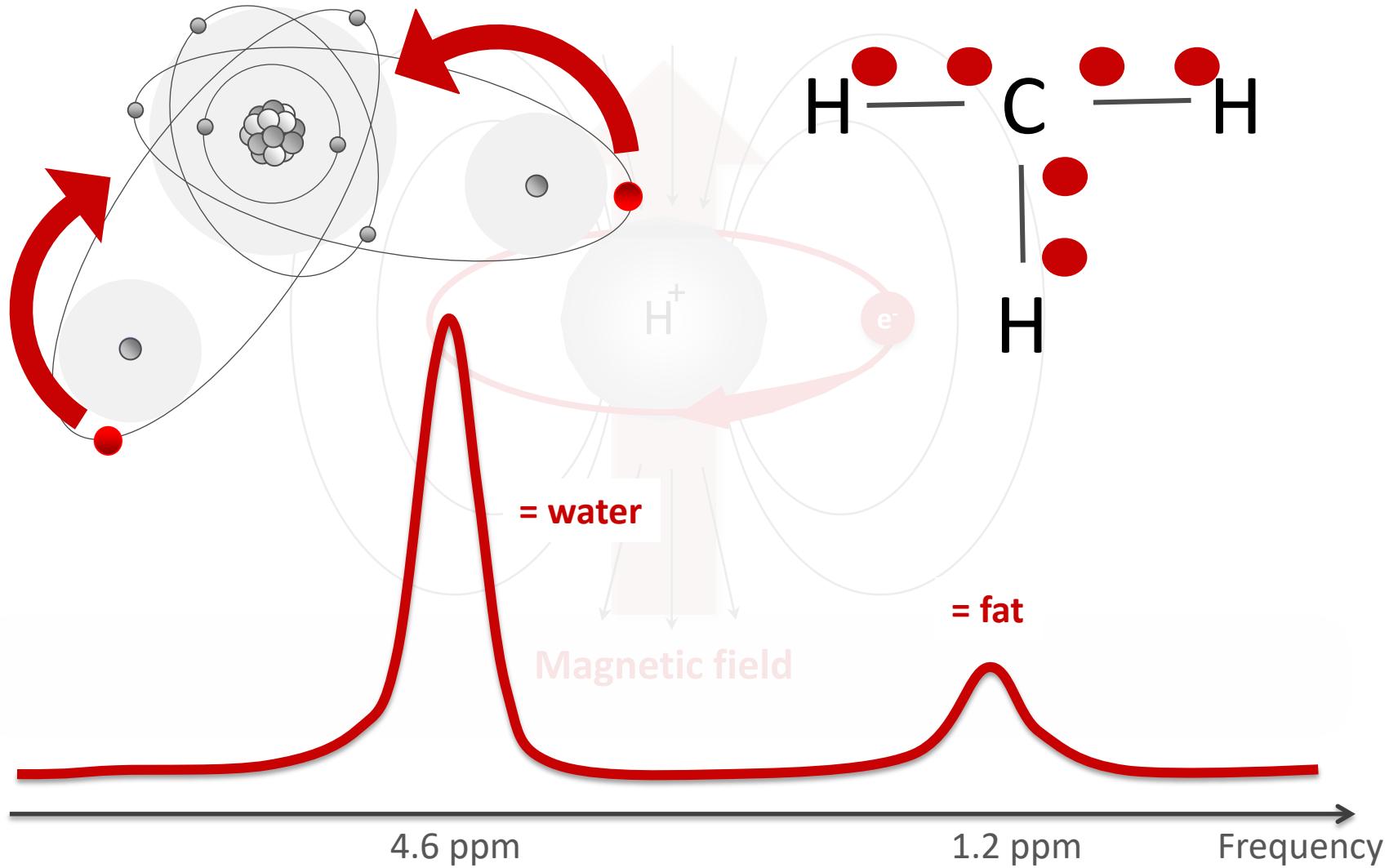
Radio receiver



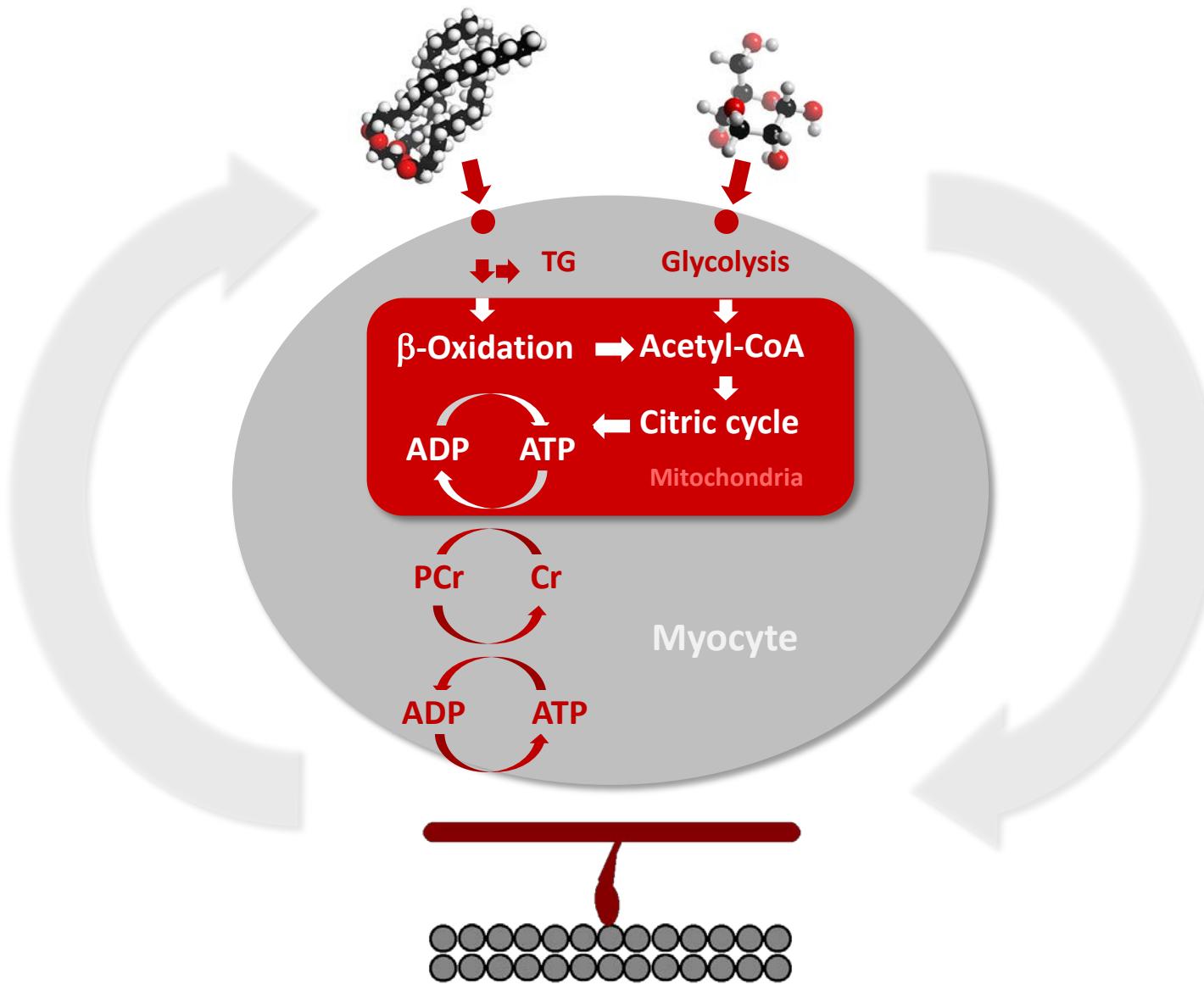
Frequency → Chemical Shift



# Chemical shift



# Cell respiration



# Spectroscopy

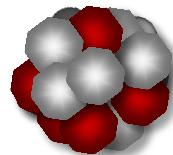
Radio transmitter



Position → Frequency



Signal source



Magnetic field



Radio receiver

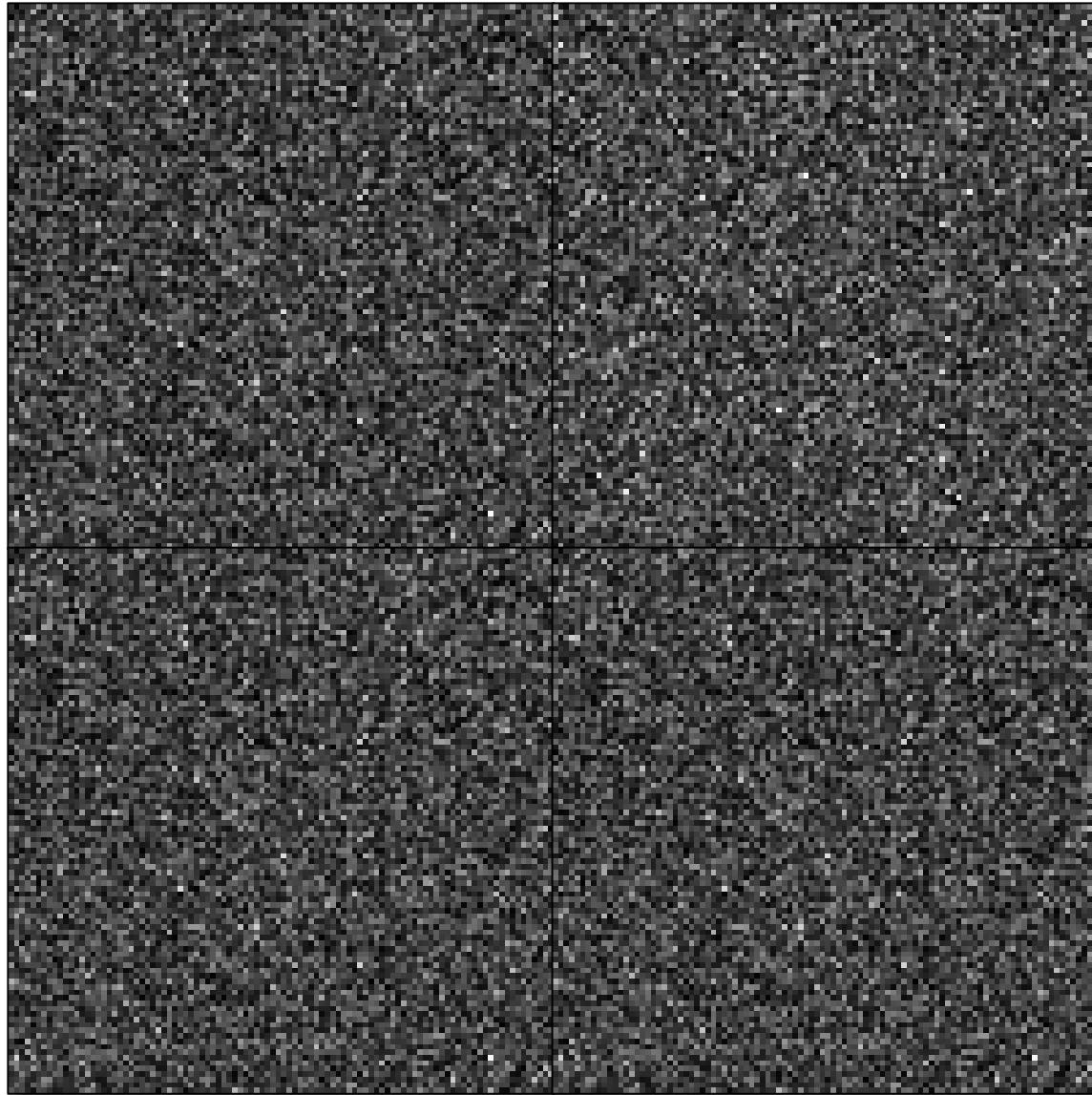


Frequency → Position



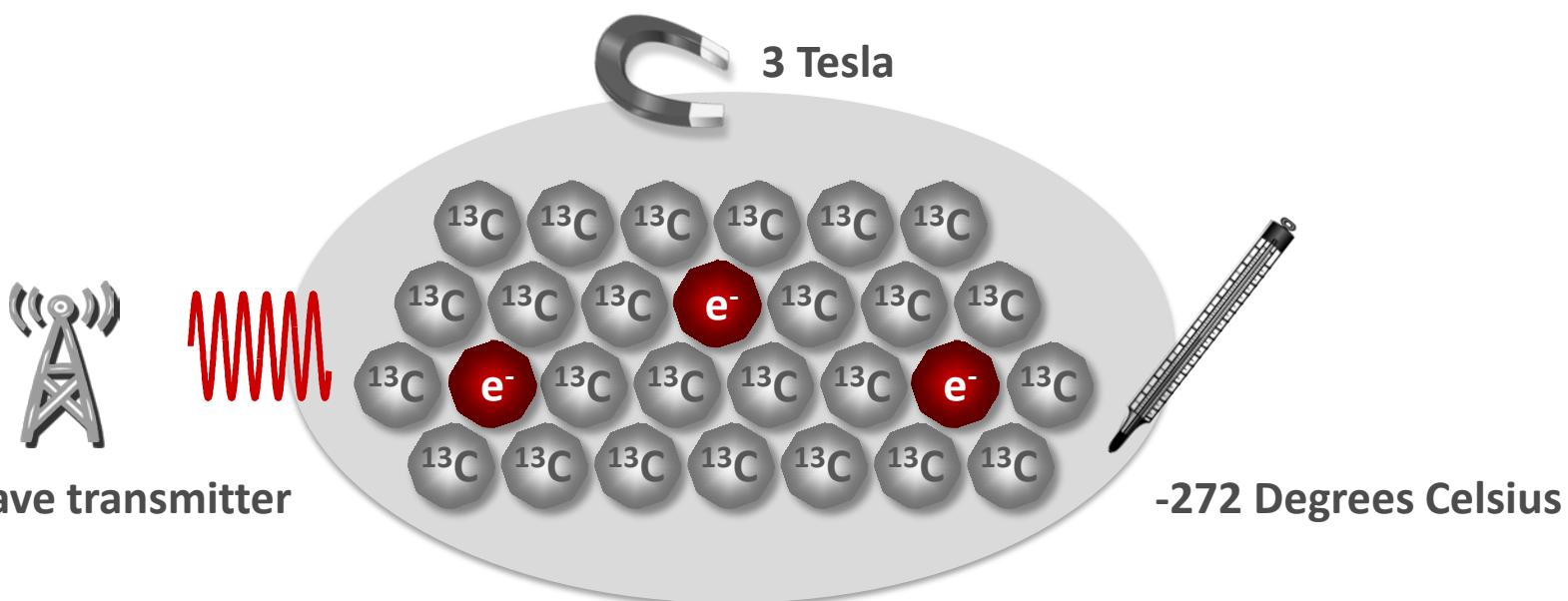
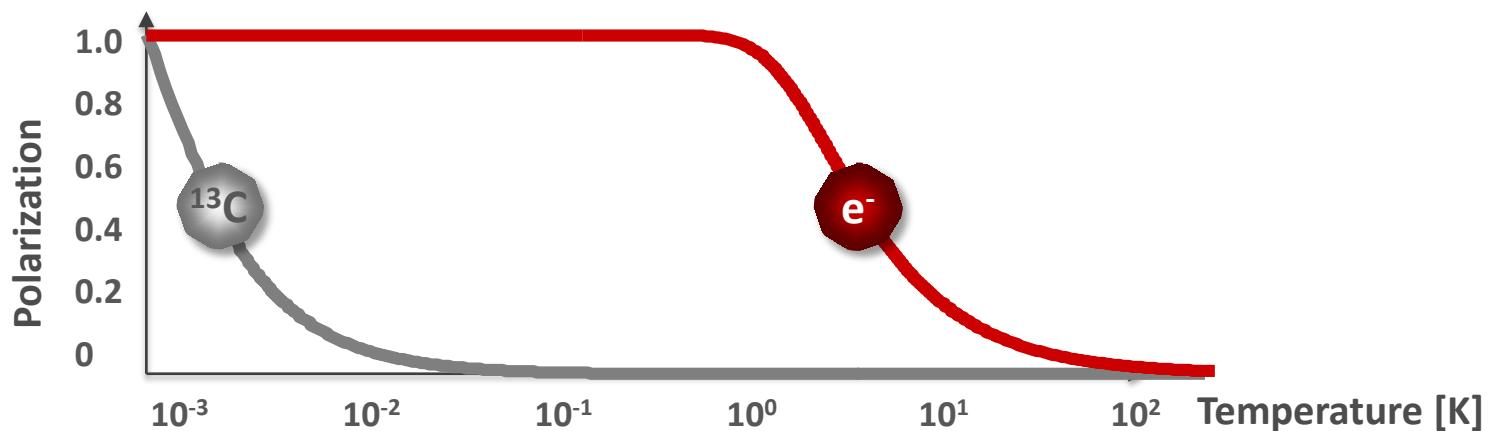
$^1\text{H}$ ,  $^{19}\text{F}$ ,  $^{31}\text{P}$ ,  $^{13}\text{C}$

# Carbon imaging

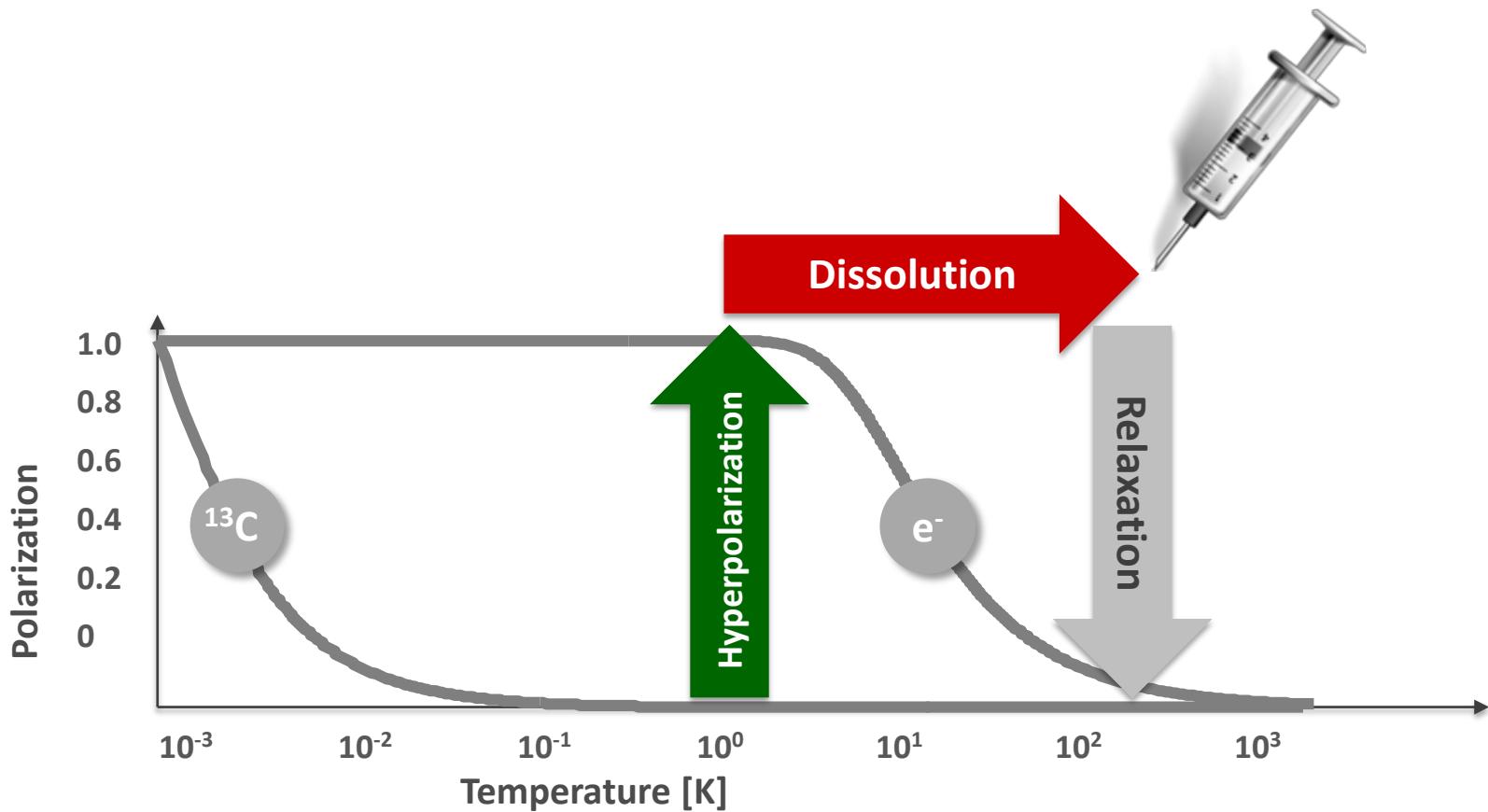


0.1-1 mM

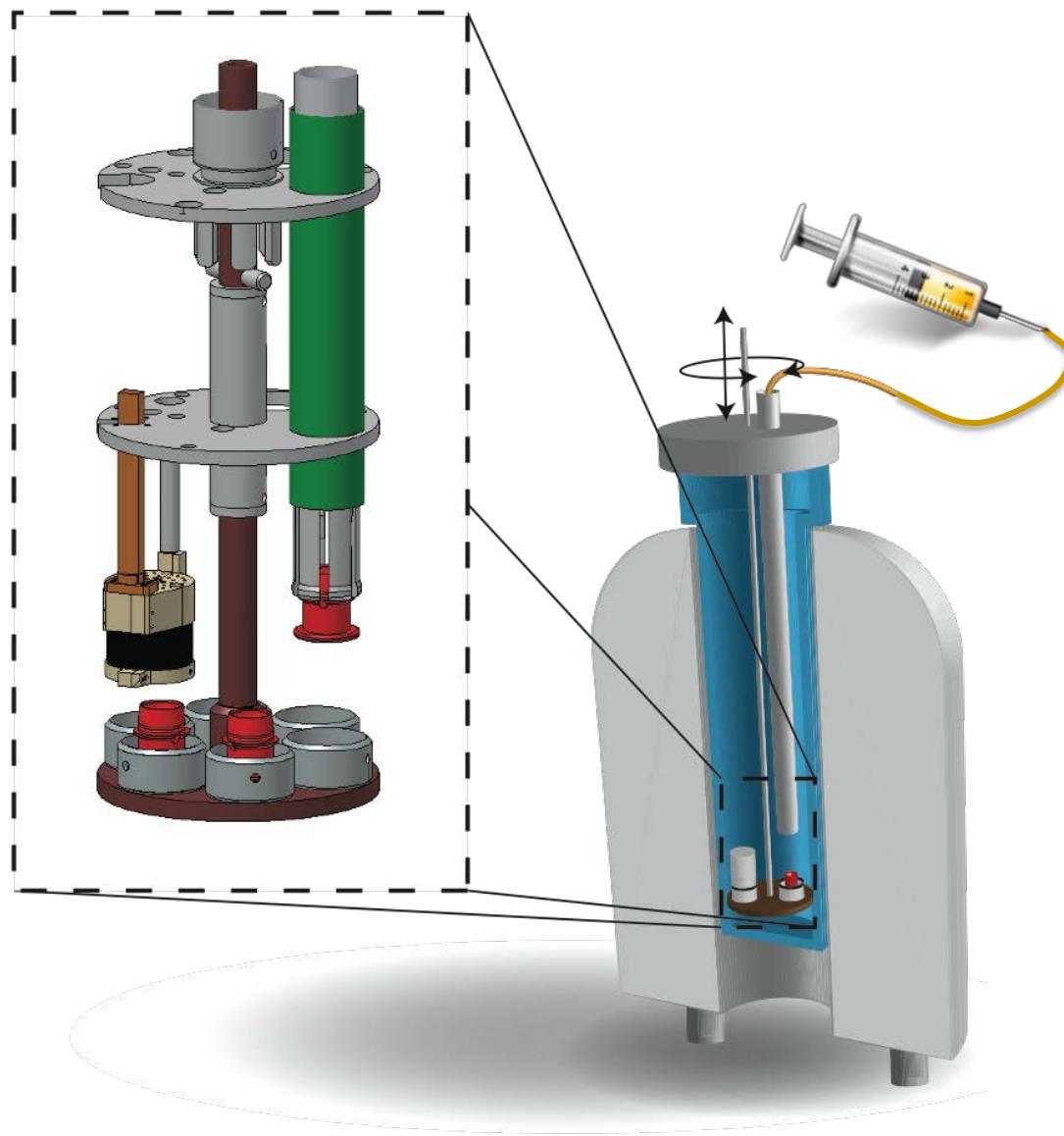
# Dynamic Nuclear Polarization



# Dissolution Dynamic Nuclear Polarization



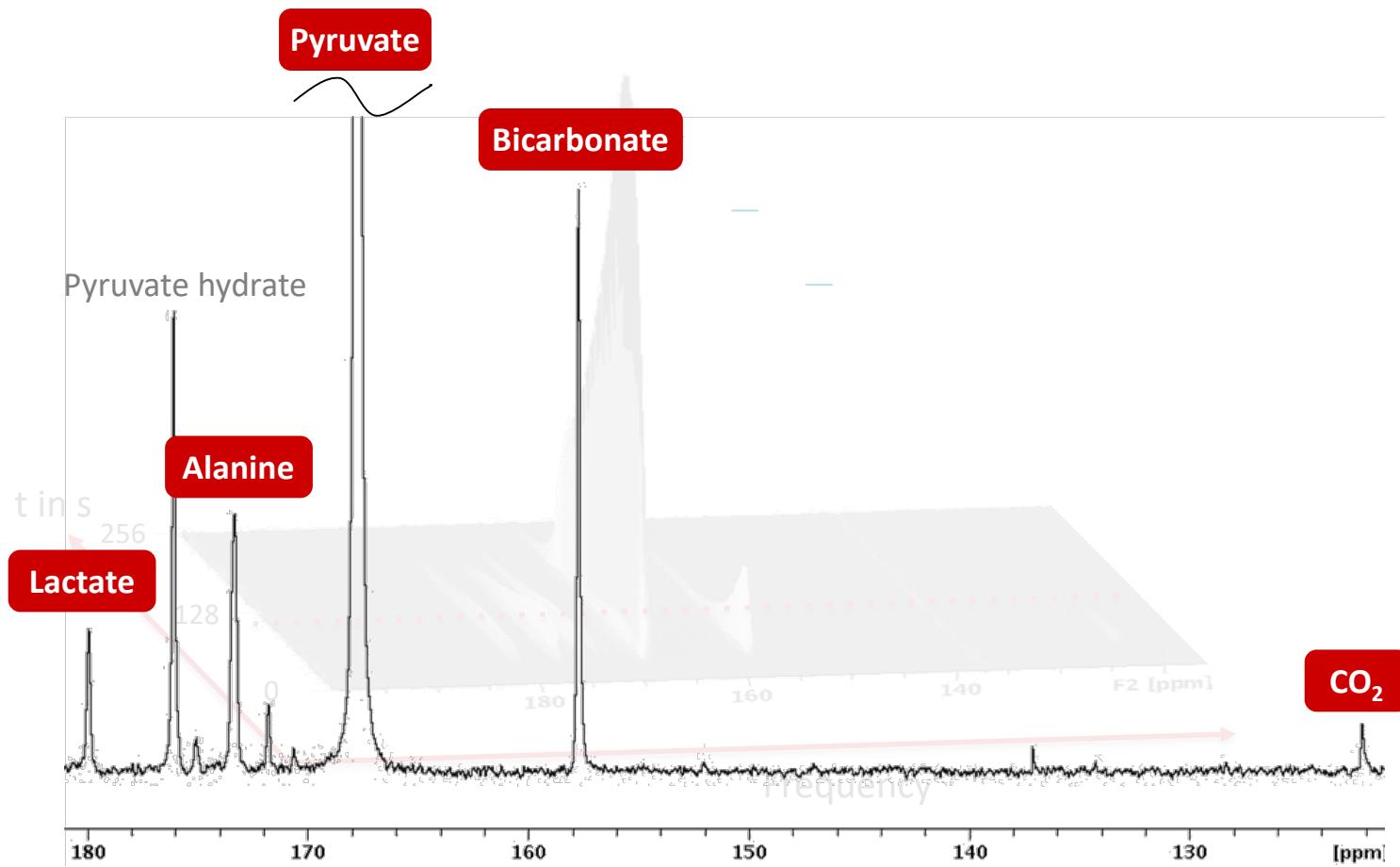
# Prototype polarizer



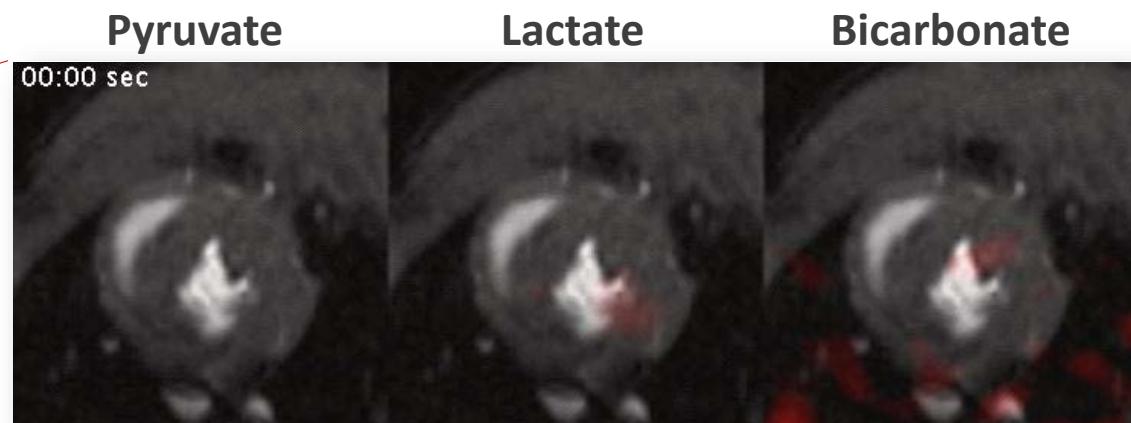
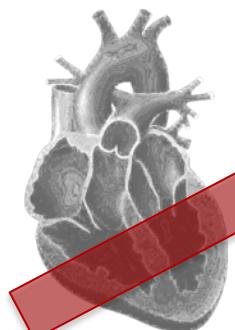
# Dissolution dynamic nuclear polarization



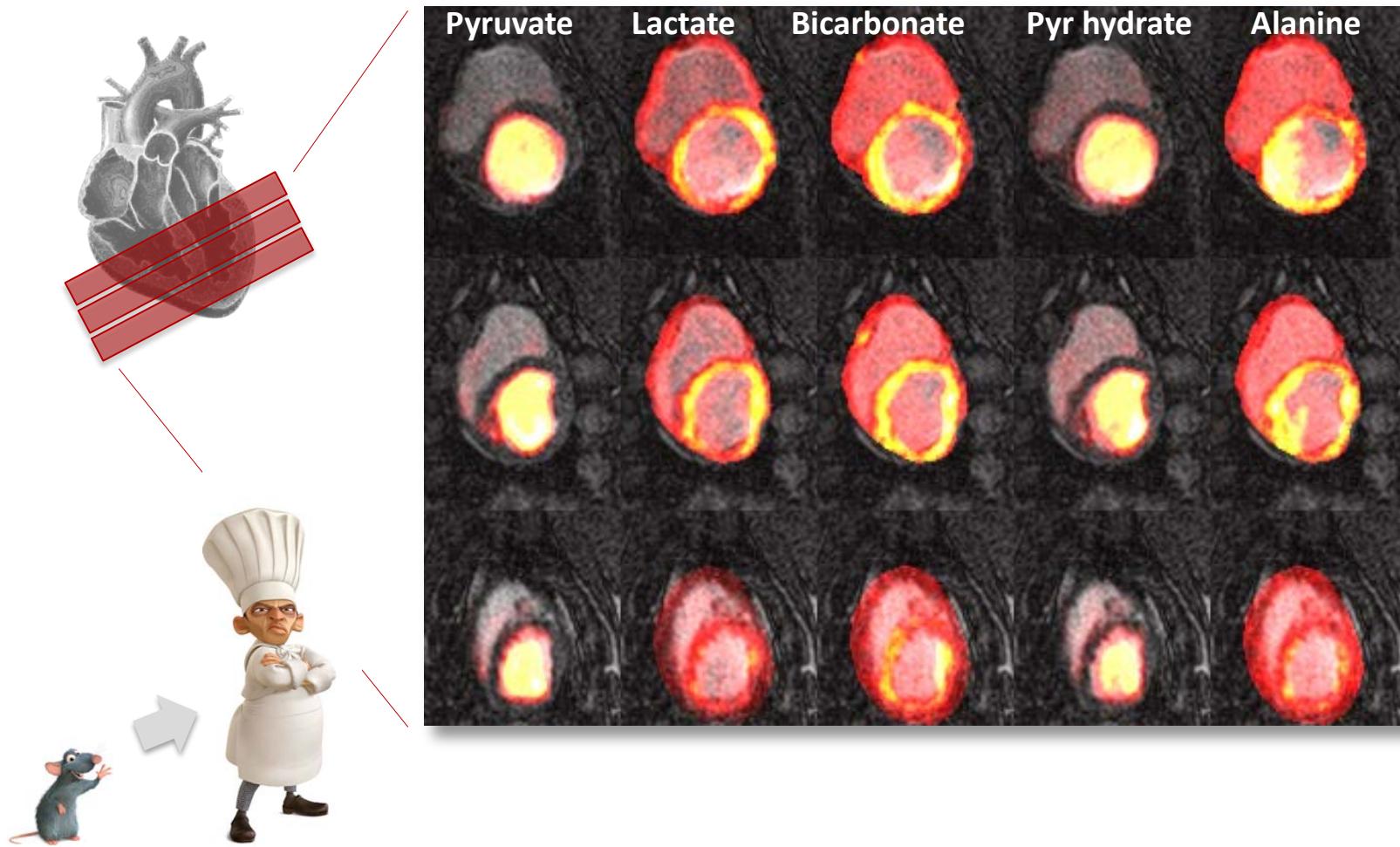
# Metabolism of isolated rat heart



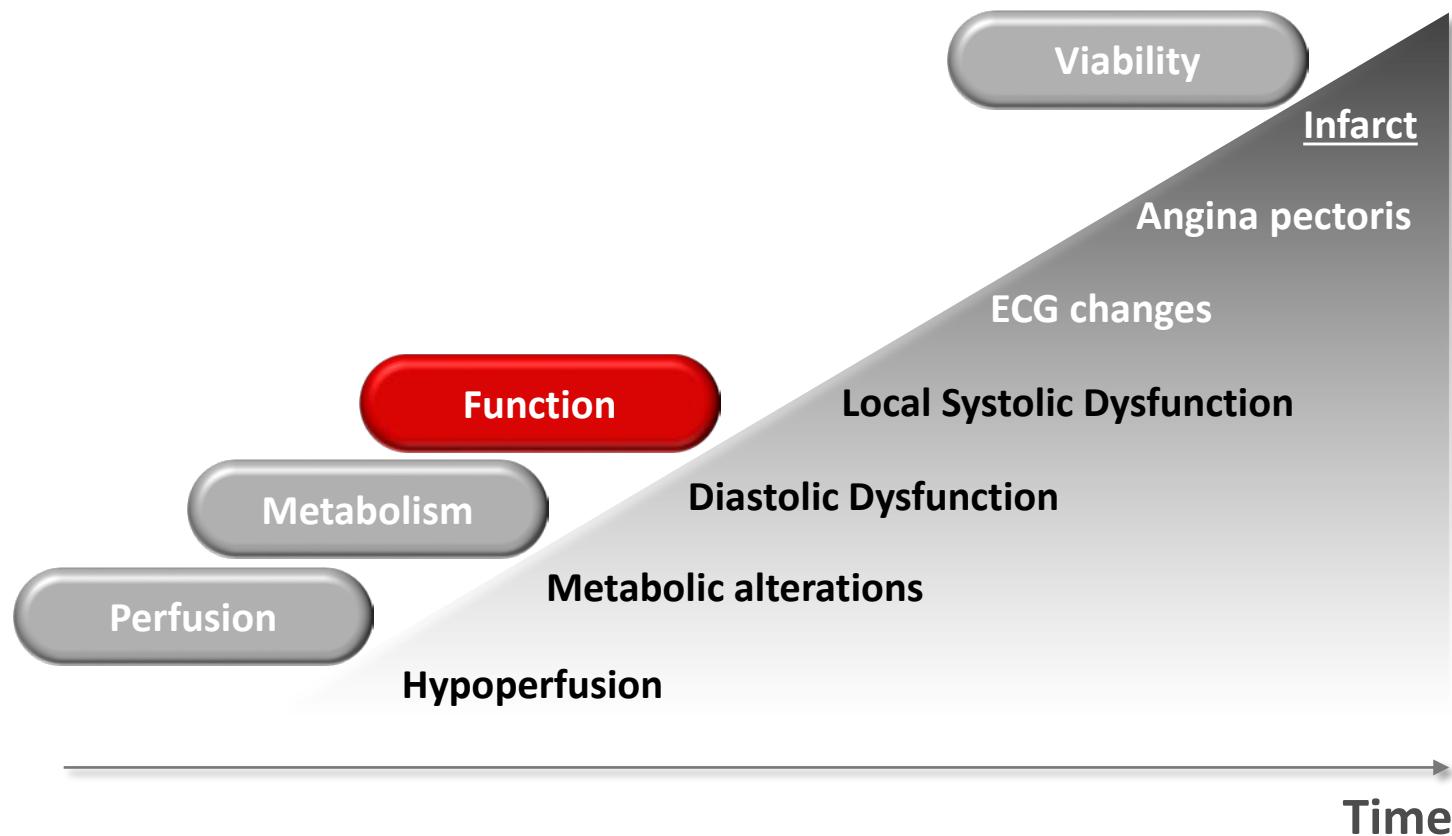
# Real-time metabolic imaging



# From bench to bedside



# Ischemic cascade



# Blood flow

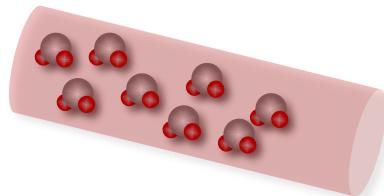
Radio transmitter



Velocity → Phase



Signal sources



Magnetic field



Radio receiver

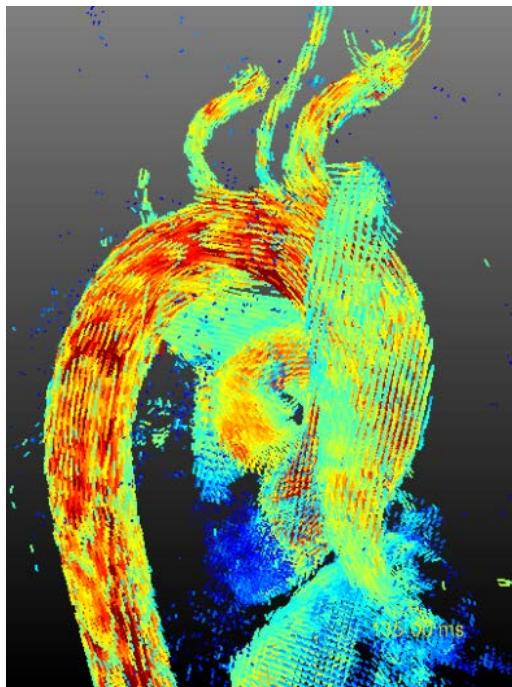


Phase → Velocity

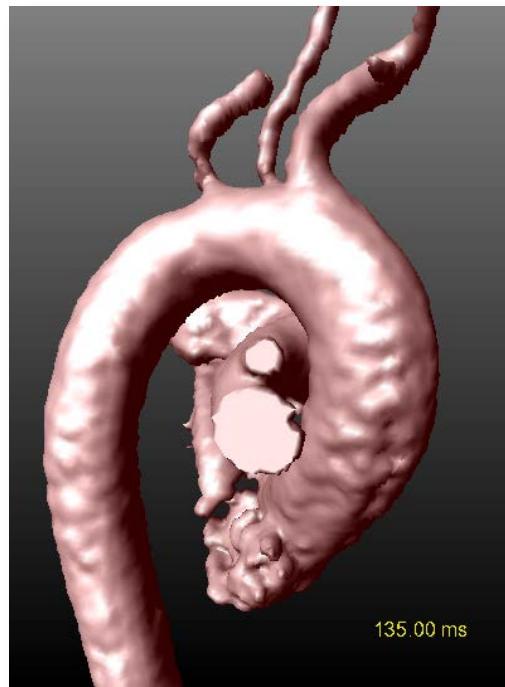


# Blood flow quantification – relative pressure maps

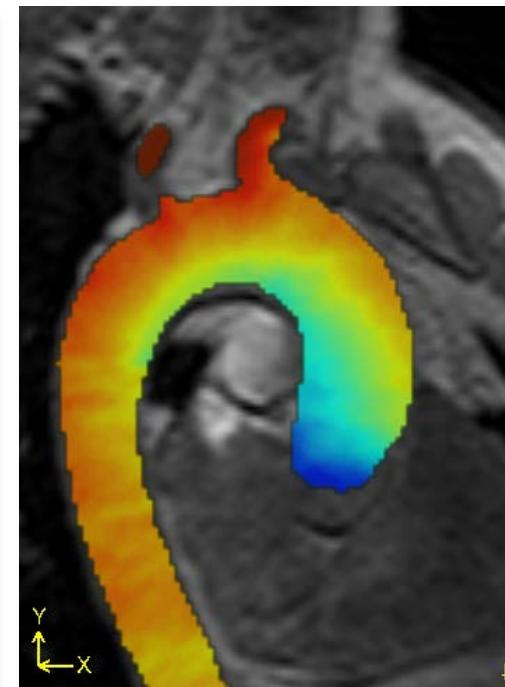
Flow field



Boundaries

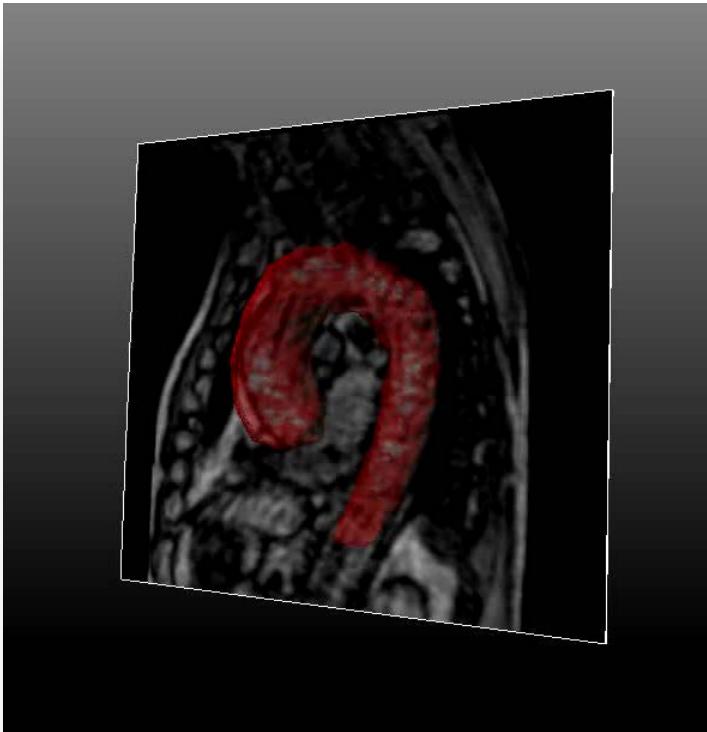


Pressure field

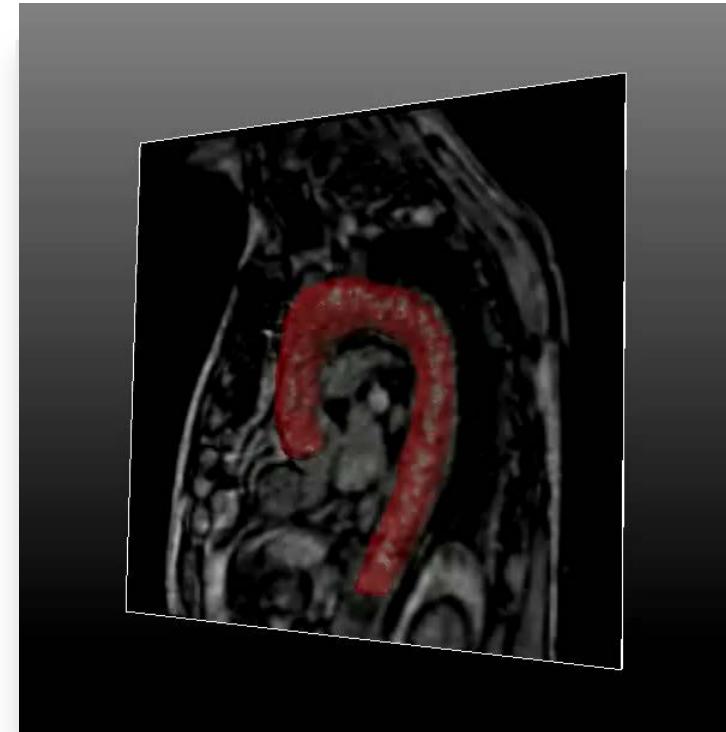


# Blood flow quantification

Pre operation

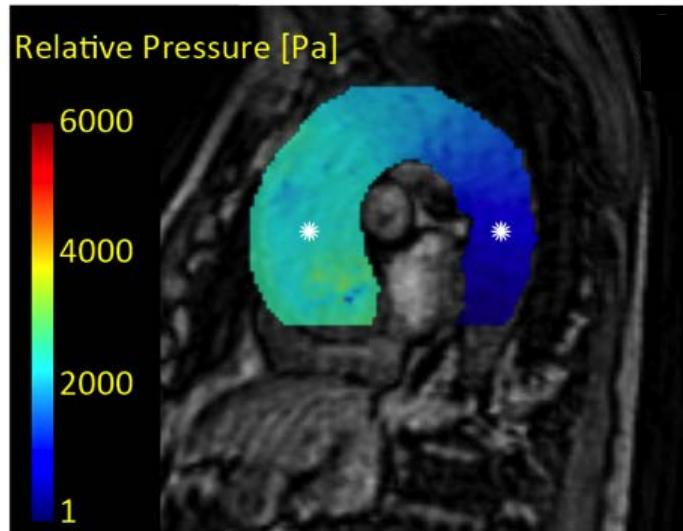


Post operation

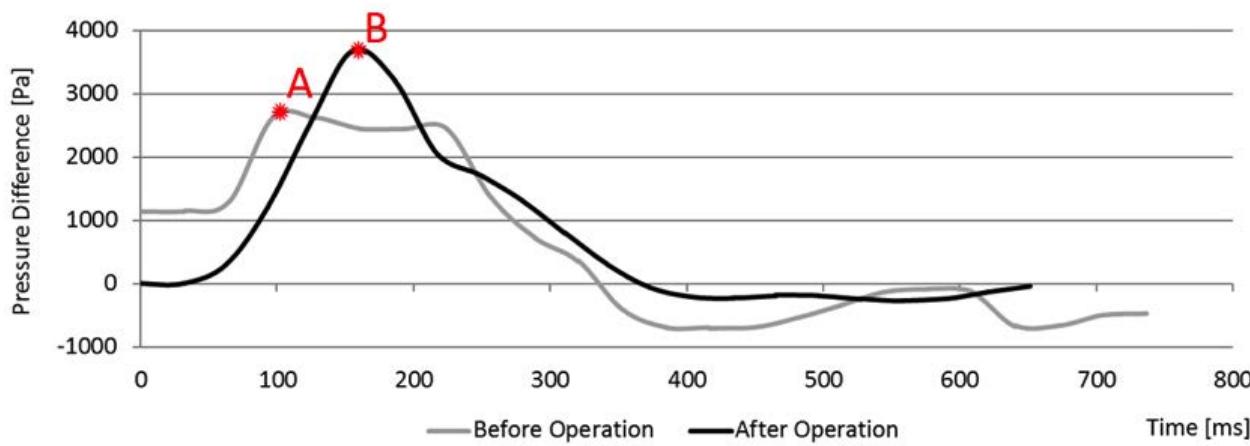
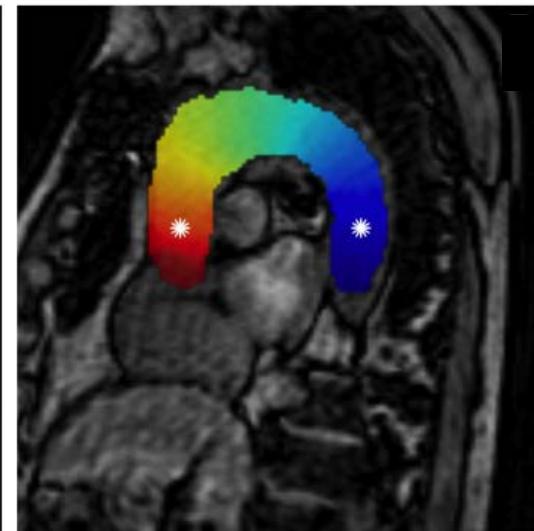


# Blood flow quantification

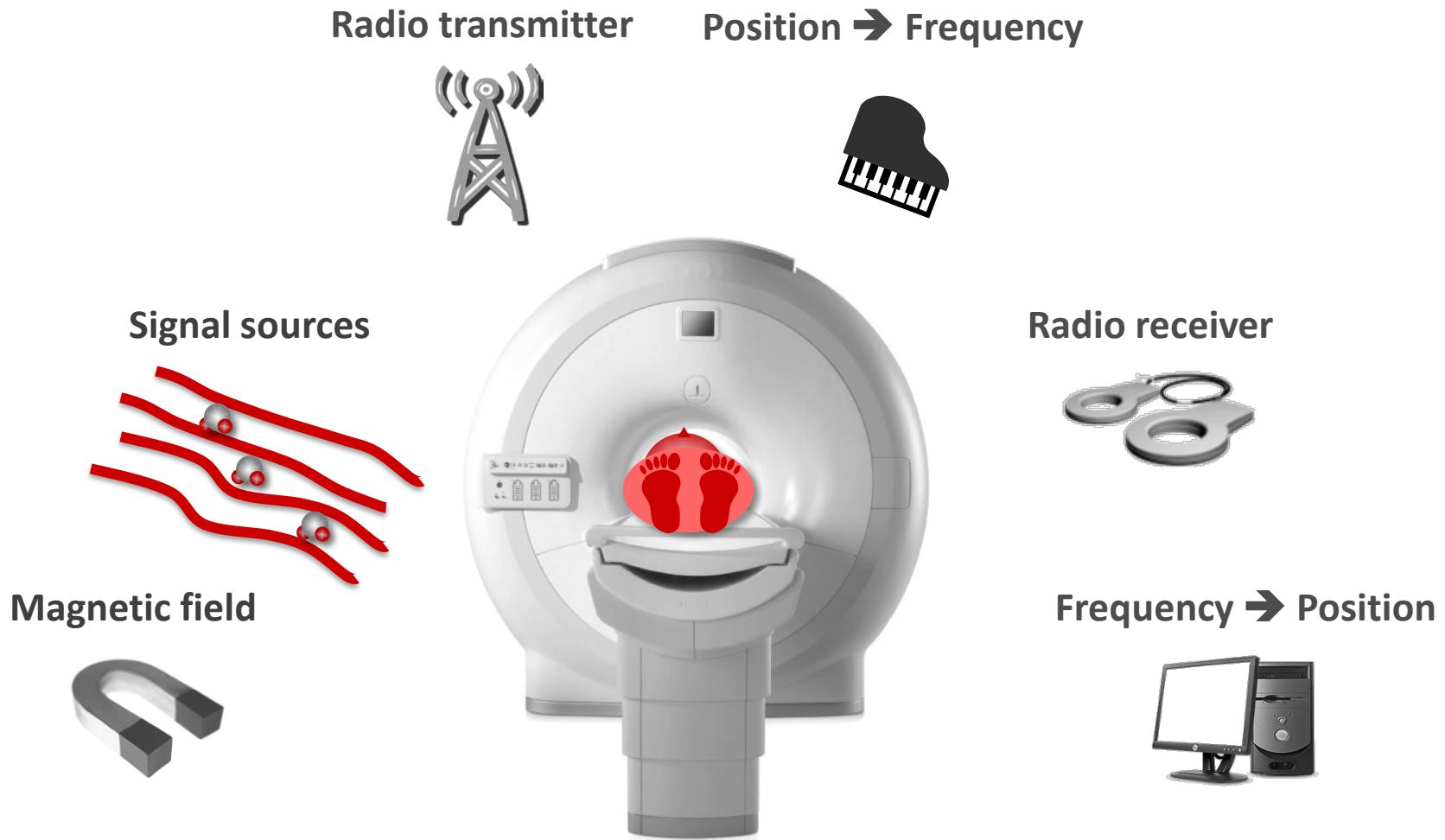
Pre operation



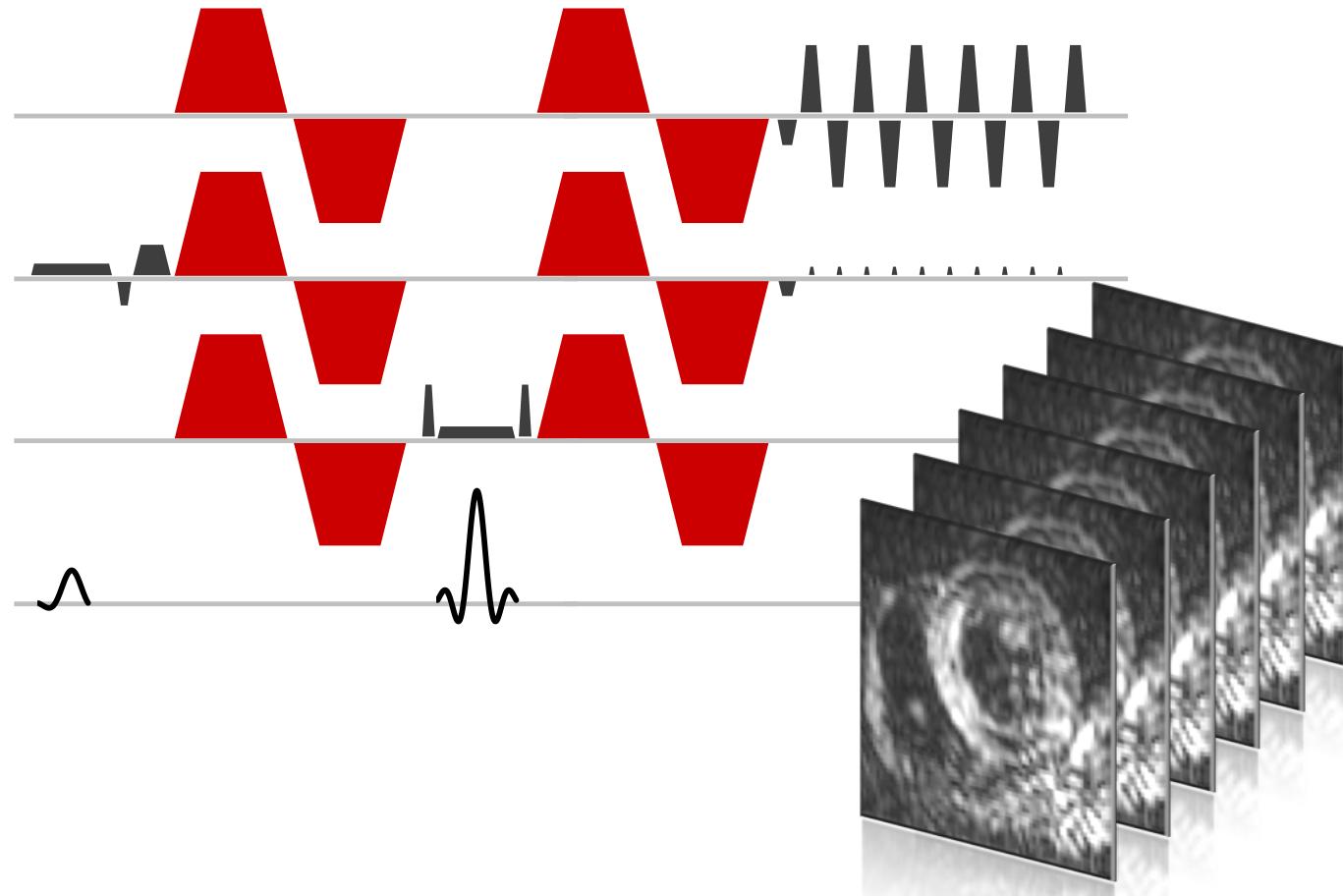
Post operation



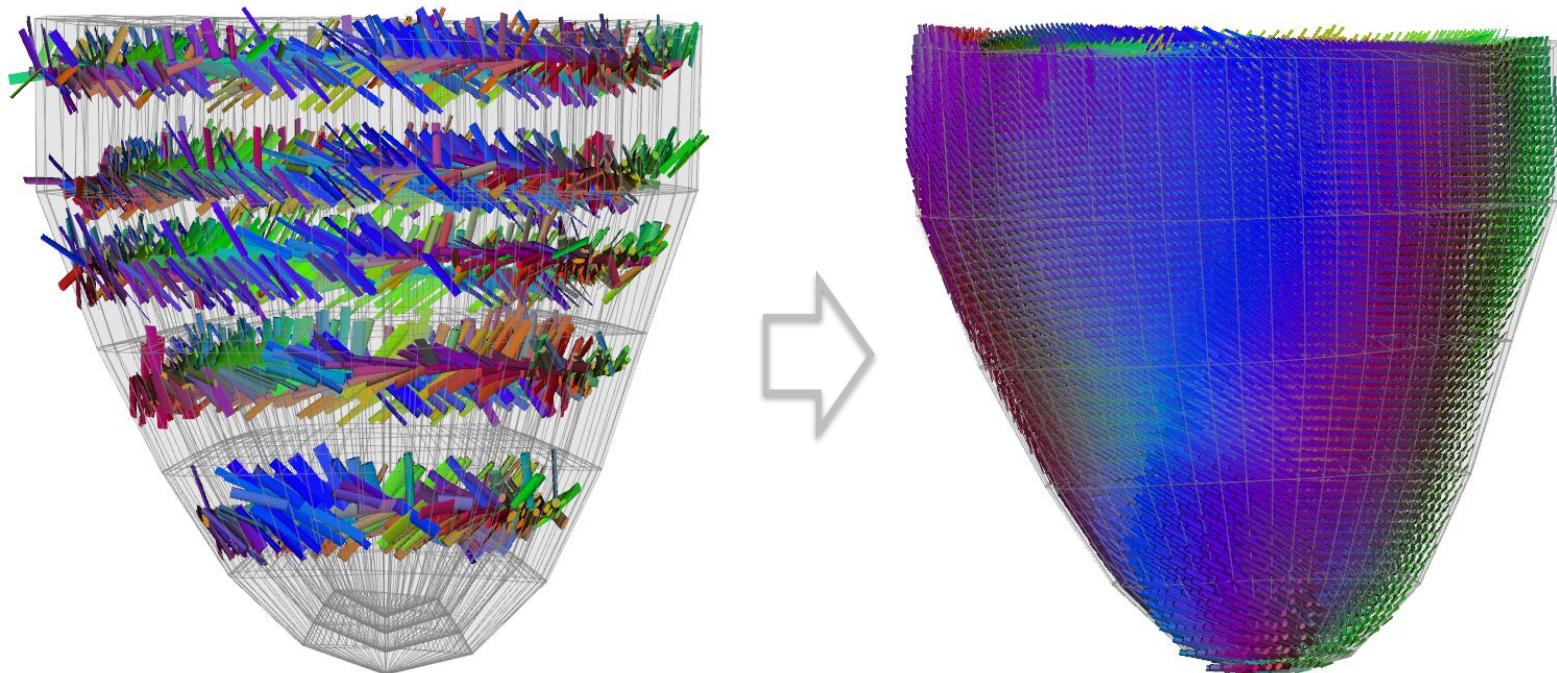
# Diffusion



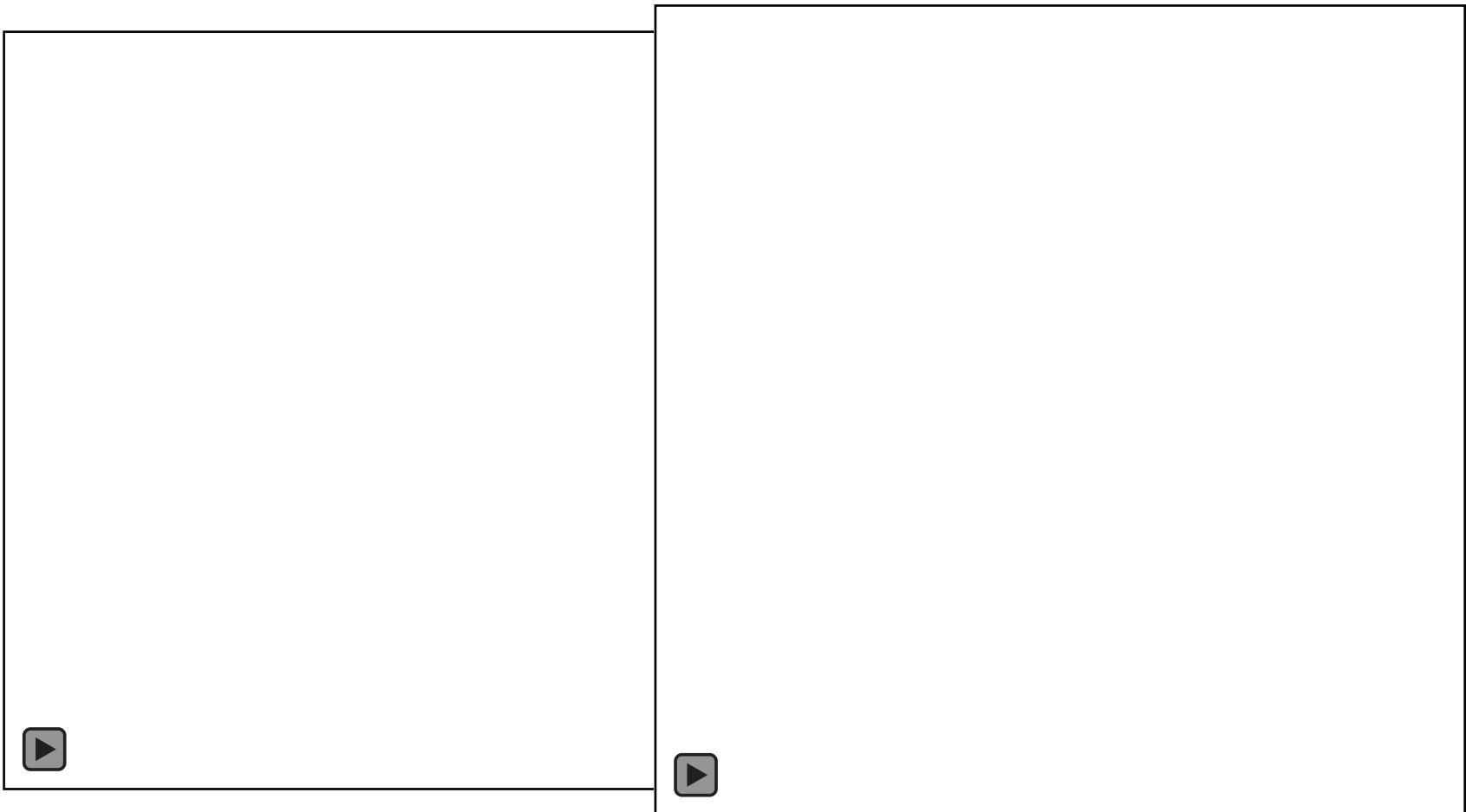
## Velocity compensated diffusion-weighted spin-echo



# Conformal mapping

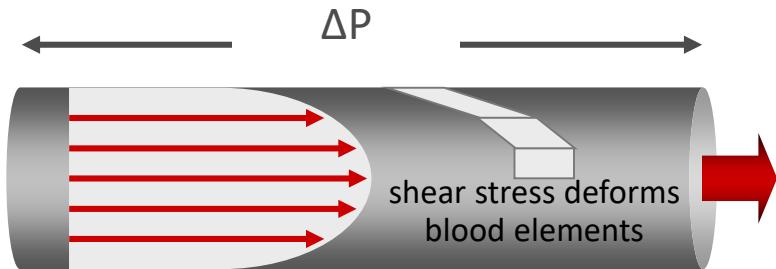


# Dynamic Diffusion Tensor Imaging

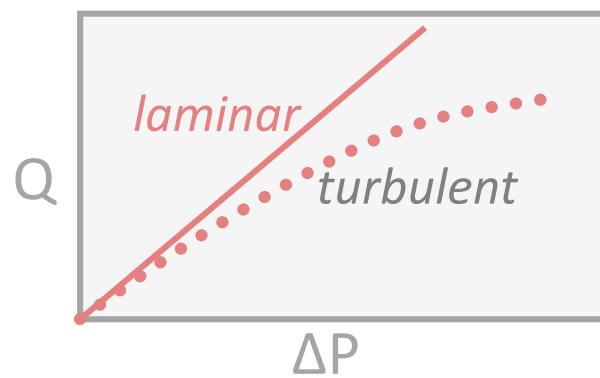
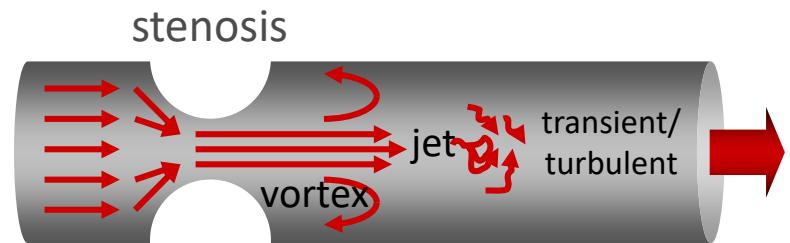


# Laminar and turbulent flows

## Laminar flow

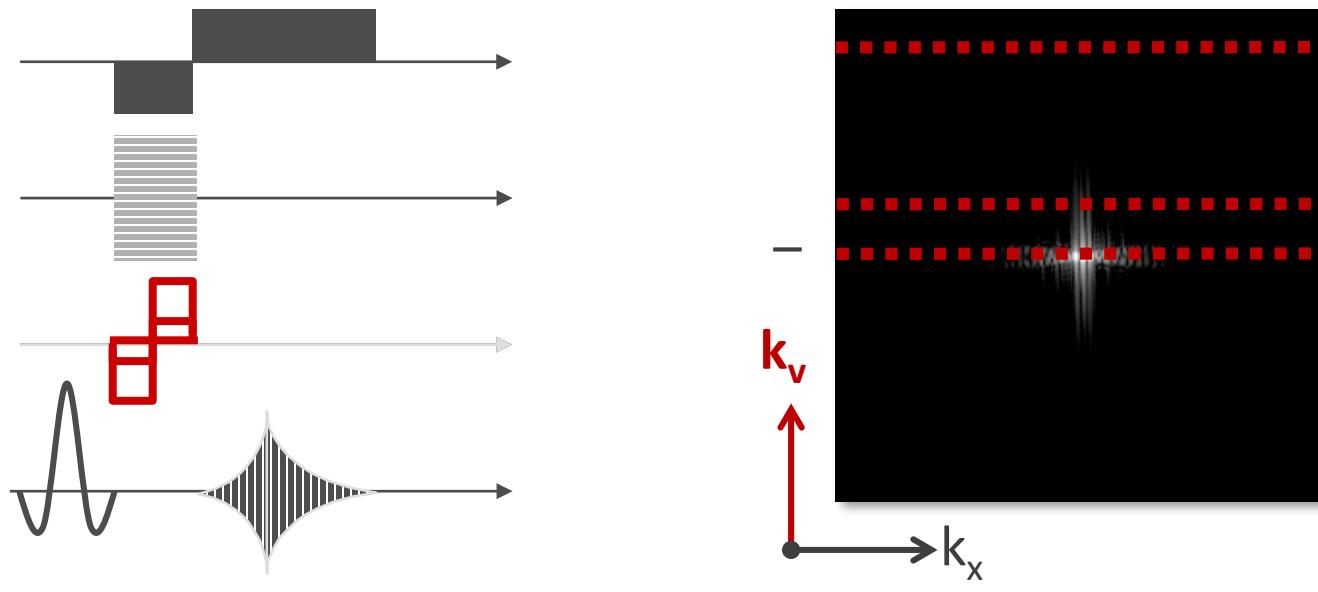


## Turbulent flow



$$\text{Reynolds Number} = \frac{\rho v d}{\mu} \propto \frac{\text{Inertial force}}{\text{Viscous force}}$$

# Universal signal model



$$S(k_v) = S_0 e^{-\frac{\sigma^2 k_v^2}{2}} e^{i(vk_v + \phi)}$$

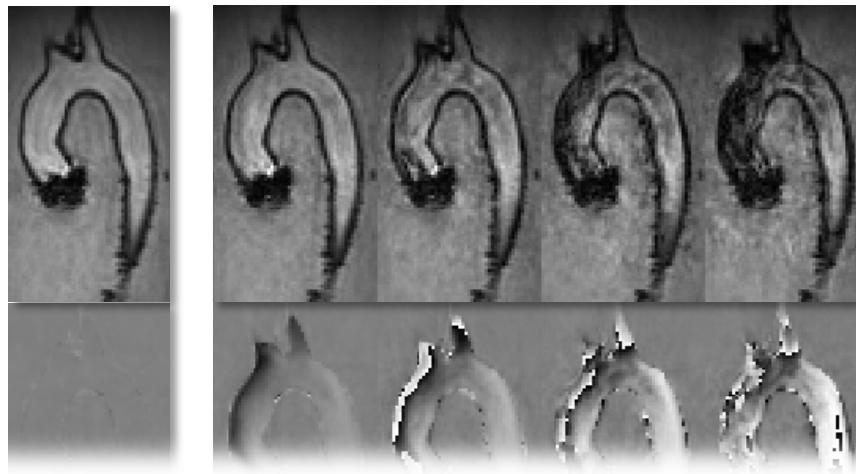
The equation represents the universal signal model:

$$S(k_v) = S_0 e^{-\frac{\sigma^2 k_v^2}{2}} e^{i(vk_v + \phi)}$$

Two images are shown below the equation, each with a red arrow pointing to a circled term in the equation:

- The left image shows a colorful, swirling pattern.
- The right image shows a brain scan with a red outline.

# Bayesian parameter estimation



$$P(\Theta, \mathbf{B} | M, I) = \frac{P(\Theta, \mathbf{B} | I) P(M | \Theta, \mathbf{B}, I)}{\cancel{P(M | I)}} \quad \Theta = [\sigma, v]$$

Incorporate bounds of parameters

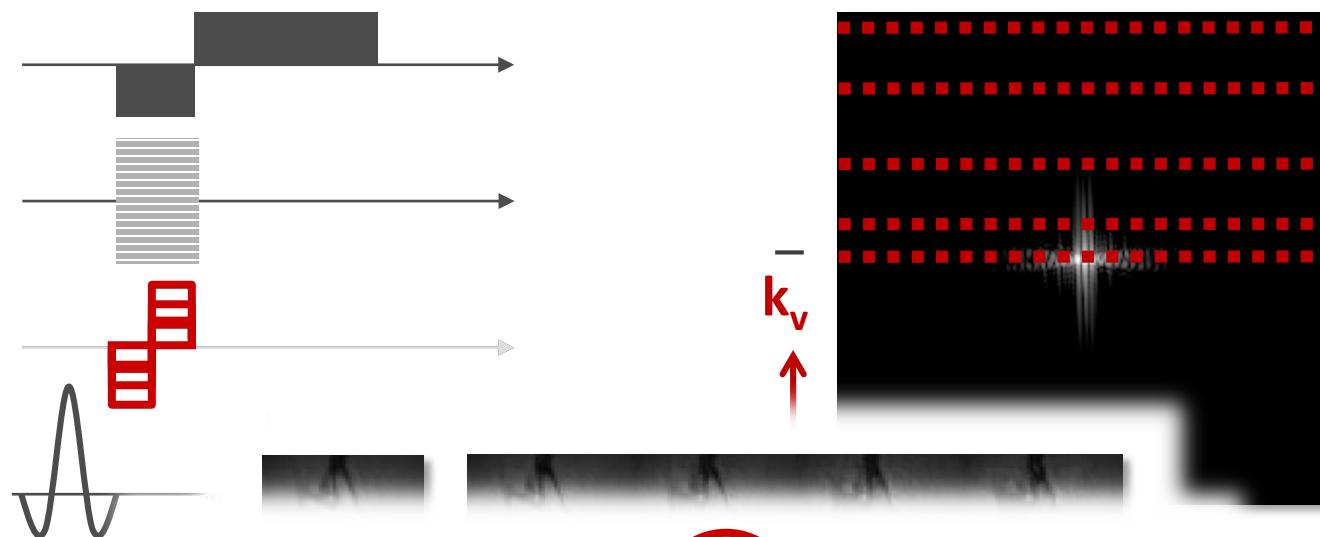
Incorporate knowledge about the noise

$$\int \int d\sigma_n dB$$

$P(\Theta | M, I)$

A large curved arrow points from the term  $\cancel{P(M | I)}$  down to the integral expression  $\int \int d\sigma_n dB$ .

# Bayesian multi-point velocity encoding



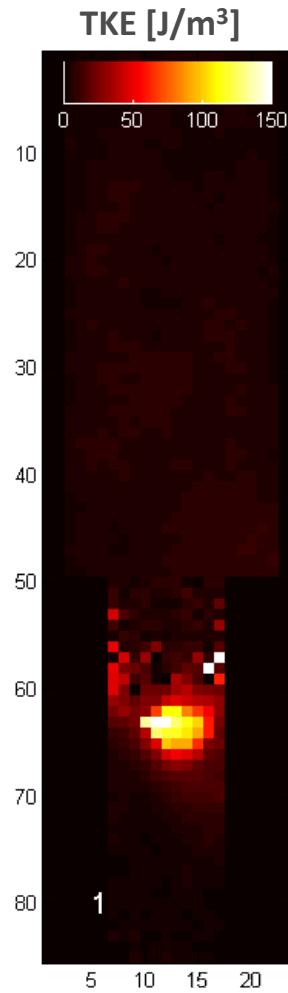
$$\mathbf{Re} = \begin{bmatrix} \delta_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \delta_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \delta_{zz} \end{bmatrix} = \rho \begin{bmatrix} \langle v_1'^2 \rangle & \langle v_1' v_2' \rangle & \langle v_1' v_3' \rangle \\ \langle v_1' v_2' \rangle & \langle v_2'^2 \rangle & \langle v_2' v_3' \rangle \\ \langle v_1' v_3' \rangle & \langle v_2' v_3' \rangle & \langle v_3'^2 \rangle \end{bmatrix}$$

$$TKE = \frac{\rho}{2} \sum_{i=1}^3 \langle v_i'^2 \rangle = \frac{\rho}{2} \sum_{i=1}^3 \sigma_i^2$$

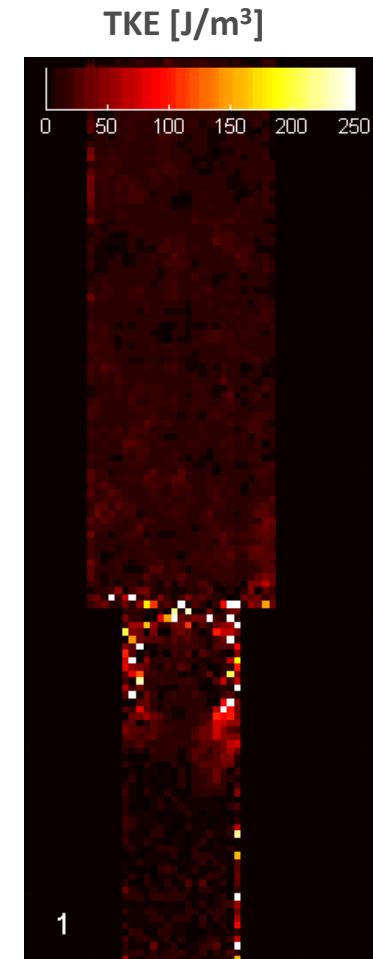
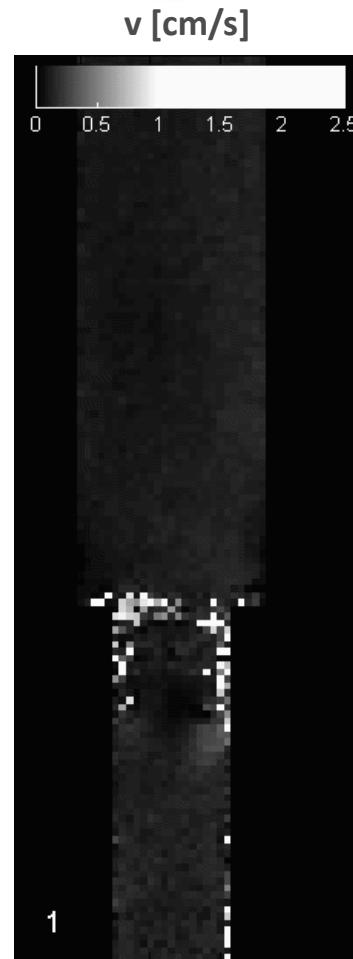
# Aortic valves in-vitro



St. Judes Medical®

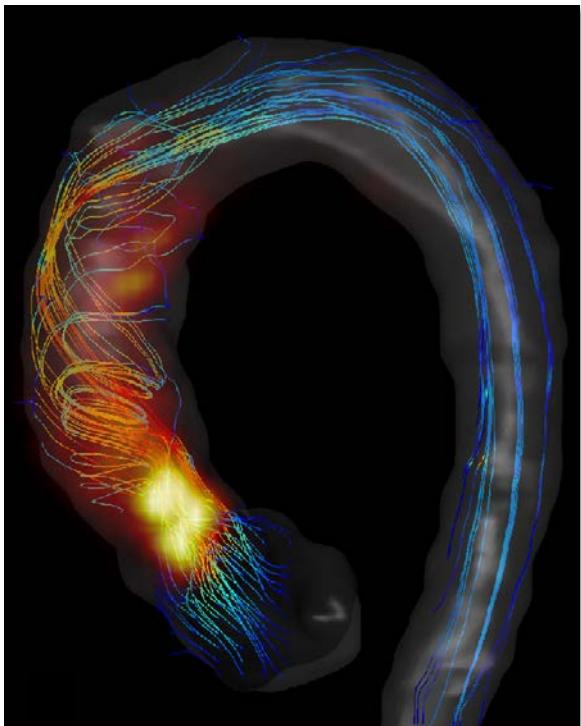


Edwards® Sapien



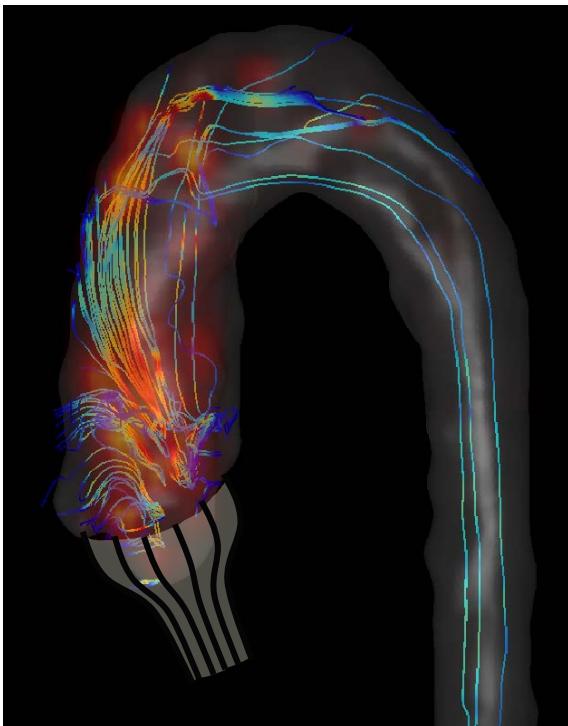
# Aortic valves in-vivo

Aortic Stenosis



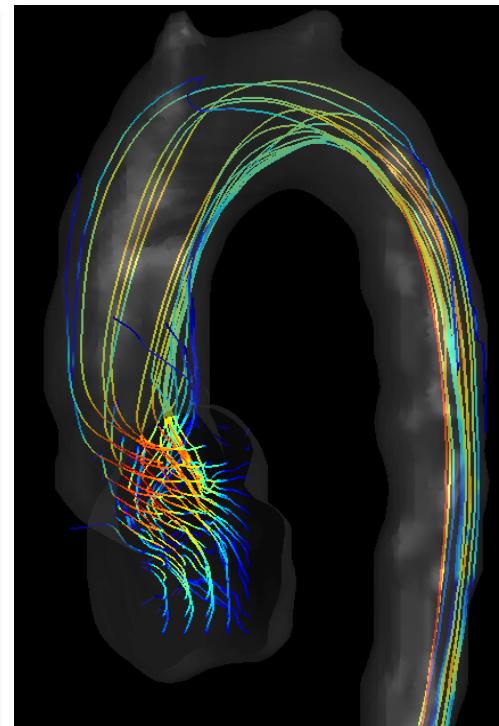
$TKE_{peak}$       **940 J/m<sup>3</sup>**  
 $V_{peak}$       **3.3 m/s**

TAVI

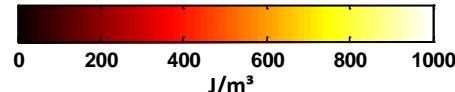
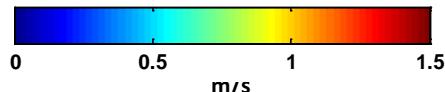


$TKE_{peak}$       **540 J/m<sup>3</sup>**  
 $V_{peak}$       **2.4 m/s**

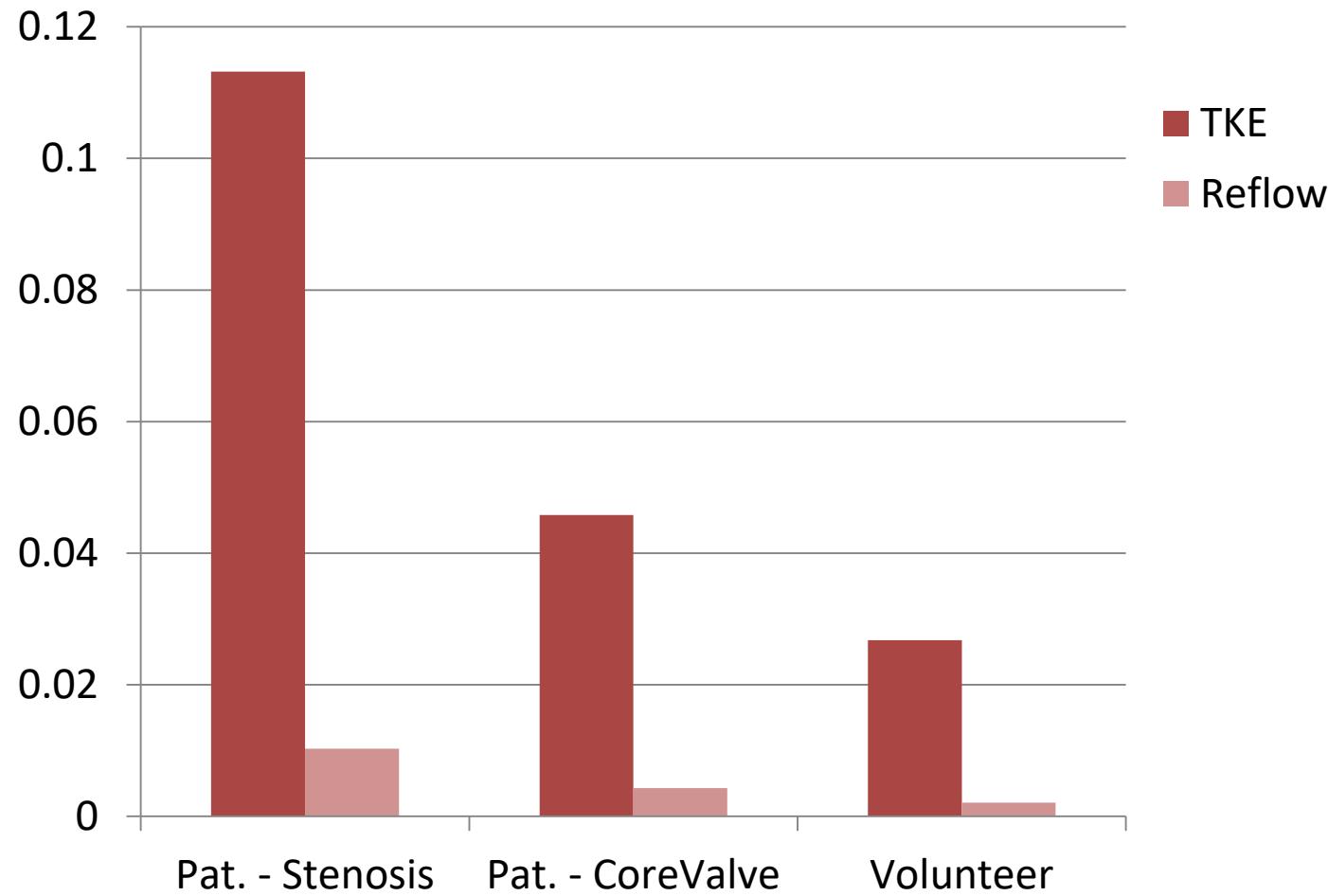
Healthy



$TKE_{peak}$       **150 J/m<sup>3</sup>**  
 $V_{peak}$       **1.3 m/s**



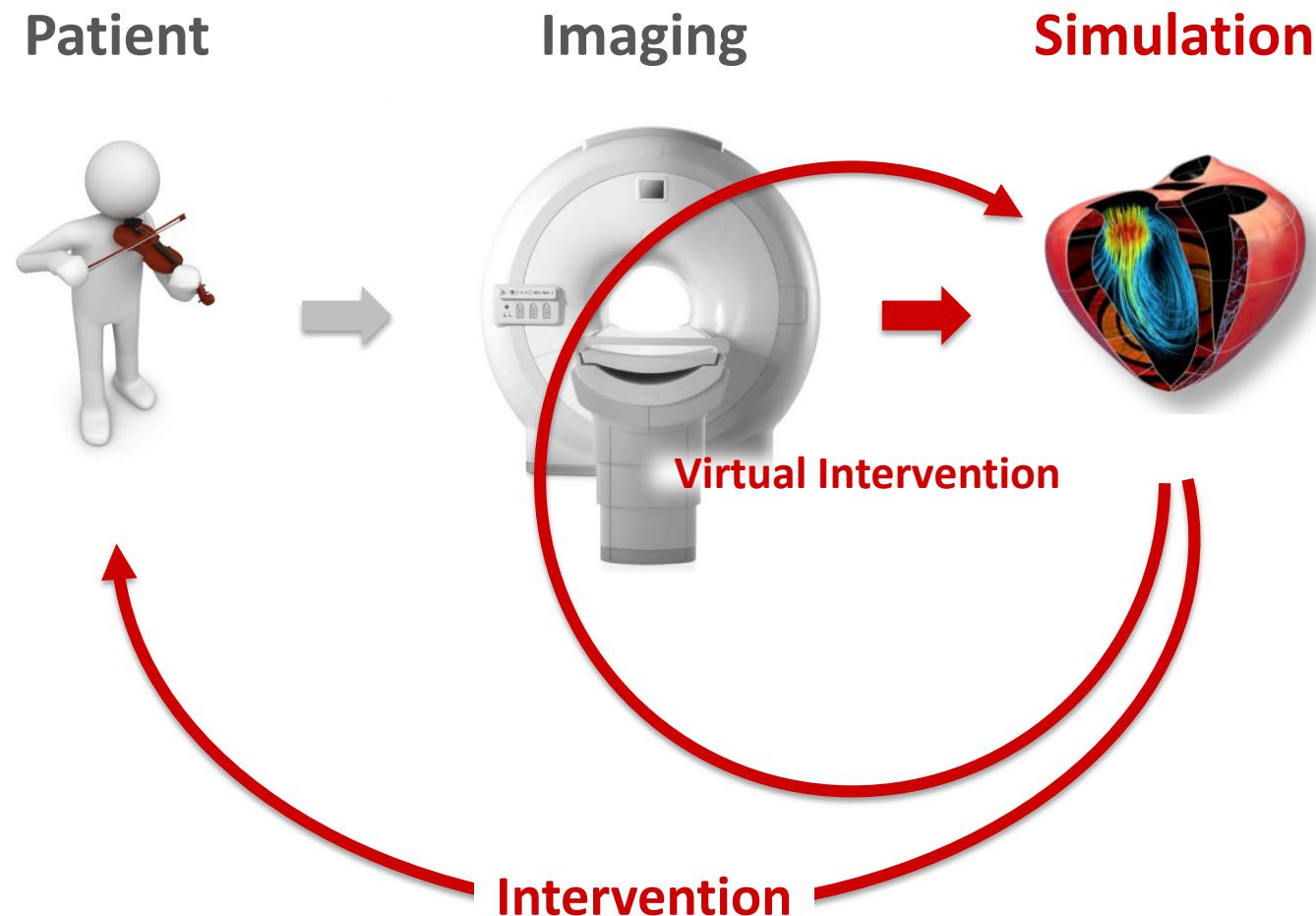
# Energy Loss index



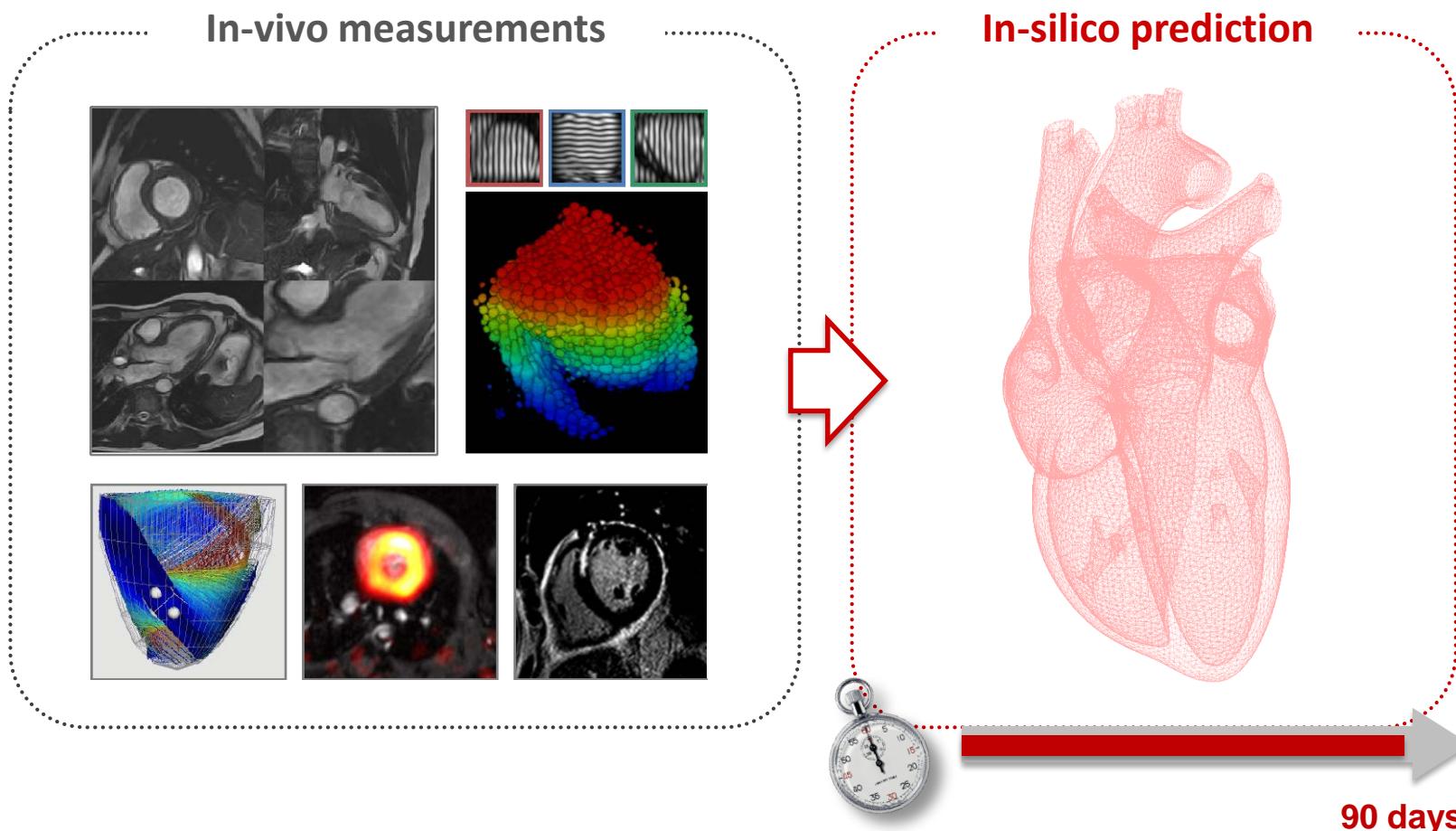
Energy Loss index

$$ELi = \left( \frac{TKE \cdot \text{voxelsize}_{z\text{-dir.}}}{MKE} + RF \right) \cdot \frac{1}{1 - RF}$$

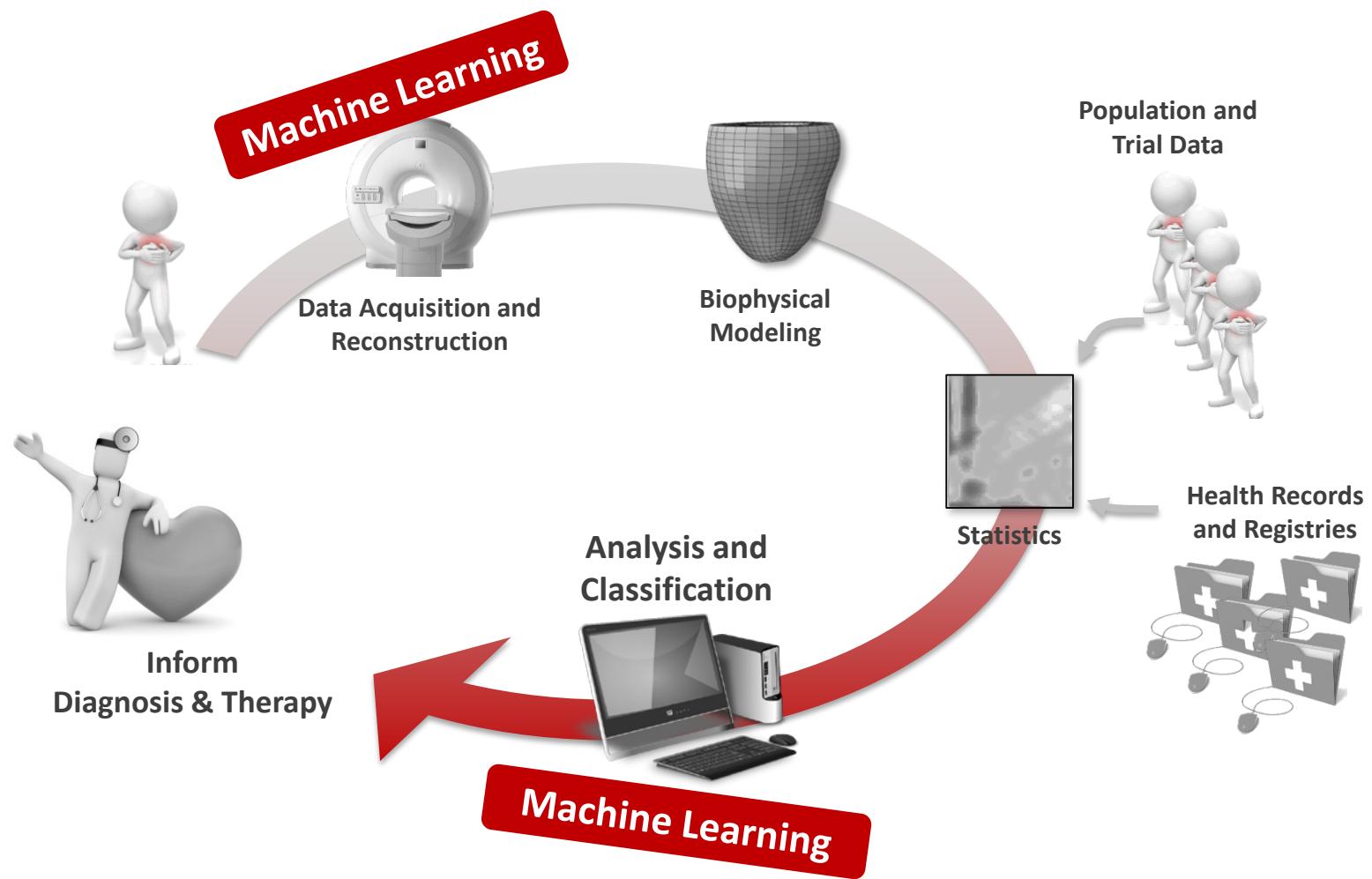
# Image guided simulations



# Image-guided modeling and prediction



# Big Data Processing



# Acknowledgements

## University and ETH Zurich

- Christian Stoeck
- Patrick Wespi
- Johannes Schmidt
- Verena Knobloch
- Max Fuetterer
- Julia Busch
- Lukas Wissmann
- Christian Binter
- Rudolf Fischer
- Michael Batel
- Marcin Krajewski
- Matthias Ernst
- Martin Buehrer
- Gerard Crelier
- Klaas Pruessmann

## King's College London

- Daniel Giese
- Claudio Santelli
- Constantin von Deuster
- Darach O'h-ici
- Nicolas Toussaint
- Sven Plein

## University Hospital Zurich

- Robert Manka
- Markus Niemann

## Funding

- SNF
- KTI/CTI
- EU FP7
- EPSRC
- PHILIPS